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The Chrono-Geometrical Structure of Special and General Relativity: towards a Background-Independent Description of the Gravitational Field and Elementary Particles.

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Abstract

Since the main open problem of contemporary physics is to find a unified description of the four interactions, we present a possible scenario which, till now only at the classical level, is able to englobe experiments ranging from experimental space gravitation to atomic and particle physics. After a reformulation of special relativistic physics in a form taking into account the non-dynamical chrono-geometrical structure of Minkowski space-time (parametrized Minkowski theories and rest-frame instant form) and in particular the conventionality of simultaneity (rephrased as a gauge freedom), a model of canonical metric and tetrad gravity is proposed in a class of space-times where the deparametrization to Minkowski space-time is possible. In them it is possible to give a post-Minkowskian background-independent description of the gravitational field and of matter. The study of the dynamical chrono-geometrical structure of these space-times allows to face interpretational problems like the physical identification of point-events (the Hole Argument), the distinction between inertial (gauge) and tidal (Dirac observables) effects, the dynamical nature of simultaneity in general relativity and to find background-independent gravitational waves. These developments are possible at the Hamiltonian level due to a systematic use of Dirac-Bergmann theory of constraints. Finally there is a proposal for a new coordinate- and background-independent quantization scheme for gravity.

I. INTRODUCTION.

After having reached a reasonable understanding of the electro-magnetic, weak and strong interactions with the $SU(3) \times SU(2) \times U(1)$ standard model of elementary particles [1] and with its subsequent tentative extensions, the main open theoretical problem is the incorporation of gravity in the framework. This is a highly non-trivial task already at the classical level, where the standard formulation of field theory in curved space-times denies any role to the concept of particle [2]. On the other hand, no one has found an acceptable blend of general relativity and quantum mechanics [3], so that it is possible that we must modify one or both the theories. The foundational problems of quantum mechanics like the *entanglement of macroscopic bodies* and the connected problem of giving a meaning to the words *measuring apparatus*, which are undefined in every approach, are still formulated at the non-relativistic level without an accepted extension to either special or general relativity due to the absence of an absolute notion of simultaneity [4]. The foundational problems of general relativity like the physical individuation of the point-events of space-time due to Einstein's Hole Argument [5] and the double role of the metric tensor, which is not only the potential of the gravitational field but also implies a *dynamical definition of the chrono-geometrical structure* of Einstein's space-times, show how far we are from a control upon relativistic causality and, in absence of realistic solutions of Einstein's equations for macroscopic bodies, from a theory of measurement involving *dynamical* and not *test* matter [6]. A naive quantization of the metric tensor destroys basic notions like *being time-, light- or space-like* which are essential for relativistic causality [7]. Theories on a background space-time like effective quantum field theory and string theory [8] have a background non-dynamical chrono-geometrical structure: as a consequence the gravitational field, instead of teaching causality to the other fields determining the null curves to be followed by massless particles, is reduced, after linearization, to a spin-2 graviton moving on a fixed light-cone like photons and gluons. On the other hand the best developed background-independent theory, loop quantum gravity [9], does not lead to a Fock space, so that no one knows how to incorporate in it the standard model of elementary particles.

This host of big unsolved problems is accompanied by a lot of other either technical or conceptual secondary problems, sometimes remnants of older ones, which, in absence of a solution, have been hidden under the rug. Here is a partial list:

- i) All the physically relevant theories are formulated in terms of singular Lagrangians.

Therefore for their Hamiltonian description and for the definition of a well posed Cauchy problem a good understanding of Dirac-Bergmann theory of constraints [10, 11, 12, 13] is needed. Moreover, a technique for the separation of the arbitrary *gauge variables* from the *gauge invariant predictable Dirac observables* has to be defined.

ii) The need of a different interpretation of the *gauge variables* in theories invariant under local inner Lie groups, like gauge theories, and in theories invariant under time or space-time diffeomorphisms, like relativistic particles, strings and every formulation of gravity.

iii) The theory of relativistic bound states [1], which requires the understanding of the instantaneous approximations to quantum field theory so to arrive at an effective relativistic wave equation and to an acceptable scalar product. In turn, the wave equation must also result from the quantization of a relativistic action-at-a-distance two-body problem in relativistic mechanics (but with the particles interpreted as asymptotic states of quantum field theory), since only in this way we can get a solution of the interpretational problems connected to the gauge nature of the *relative times*, a problem going back to Tetrad and Fokker [14], not to speak of Droste [15] for the beginning of the still unsolved general relativistic two-body problem, and lying at the basis of the interpretational problems with the Bethe-Salpeter equation [16].

iv) The *problem of time* in general relativity [17, 18]: Mach's influence on Einstein resulted in a preference for compact space-times without boundary and, then, in the Wheeler-DeWitt interpretation implying local evolution in an internal either extrinsic (York) or intrinsic (Misner) time but also a frozen formalism with no evolution in the reduced phase space [19]. On the other hand, in non-compact space-times the many-fingered time notion of evolution [18] was introduced as the general relativistic counterpart of the Tomonaga-Schwinger special relativistic approach to quantum field theory [20]. However, already in special relativity, the Torre-Varadarajan no-go theorem [17] shows that there is a ultraviolet obstruction to its implementation at the quantum level.

v) The claimed superiority of the configurational manifestly covariant approach over the Hamiltonian approach requiring an explicit definition of *time* through a 3+1 decomposition of space-time. However, this illusion is broken by the necessity of a well posed formulation of the Cauchy problem to identify which are the *predictable* quantities.

vi) The necessity of the Hamiltonian approach to study: a) the notions of relativistic center of mass and relative variables for the relativistic N-body problem and for relativistic fluids and fields; b) the impossibility, in the rest frame, to make a unique separation of global

rotations from vibrations due to the non-Abelian nature of the angular momentum Noether constants of motion both at the non-relativistic and relativistic levels (as a consequence Machian concepts may be consistently introduced only for zero angular momentum); c) how to recover notions like the tensor of inertia, which, together with the Jacobi coordinates, exist only in Galilei space-time, by means of relativistic multipolar expansions.

vii) Due to the absence of absolute simultaneity, the 3+1 decomposition of space-time is fundamental both in special and general relativity to introduce well defined *equal time Cauchy 3-spaces* and notions of spatial distance and one-way velocity of light. The lack of a good notion of simultaneity, generalizing Einstein's convention for clock synchronization in inertial frames, is at the basis of pathologies like the coordinate singularities appearing in rotating frames (the rotating disk, the Sagnac effect; see the bibliography of Refs.[21, 22]) and in the Fermi coordinates of a non-inertial observer.

viii) The lack of an accepted theory of the measurements made by non-inertial observers and of which are the *relativistic inertial forces* seen by them Ref.[22]. The problem of the description of gyroscopes in space experiments as pole-dipole systems and the need to replace metric gravity with tetrad gravity (a theory of non-inertial observers) when fermions are present.

Therefore, we felt the exigence of a revisitation of both special and general relativity at the classical level to see whether it is possible to arrive at a scenario with a unified description of particle physics and gravitational field in some suitable class of non-compact space-times allowing, when we switch off the Newton constant, a deparametrization to a description of particle physics in Minkowski space-time in all those general non-inertial frames where it is possible to define a good notion of simultaneity. It is hoped that this scenario will allow to arrive to a new quantization scheme of the four interactions in a way compatible with relativistic causality and the dynamical chrono-geometrical structure of space-time.

Here we delineate the general framework of this research program and its status after various recent achievements. After a review of constraint theory, we discuss the problem of the admissible notions of simultaneity in special relativity, where only the chrono-geometrical structure of Minkowski space-time is absolute, and its treatment according to the 3+1 point of view, leading to parametrized Minkowski theories for the description of every isolated system. The associated special relativistic general covariance of this theory shows that all such notions of simultaneity are *gauge equivalent* and that there is a preferred intrinsic notion leading to the *rest-frame instant form of dynamics*. In Ref.[23] there is a complete review of

all the special relativistic systems which have been treated in this way. Then we identify a model for metric and tetrad gravity in a suitable class of non-compact space-times, in which the admissible 3+1 splittings tend to space-like hyper-planes at spatial infinity in such a way that the deparametrization of general relativity with matter leads to the rest-frame instant form description of the same matter in Minkowski space-time. Since in general relativity the chrono-geometrical structure of the space-time is dynamical, we find that the admissible notions of simultaneity, besides being gauge equivalent like in special relativity, are now *dynamically* determined by the solutions of the Hamilton-Dirac equations, equivalent to Einstein's equations.

II. CONSTRAINT'S THEORY

Most of the physically relevant systems are described by means of singular Lagrangians. This means that the Hessian matrix, whose elements are the second derivatives of the Lagrangian with respect to the velocities, has zero eigenvalues. This implies that

i) the action is (quasi-) invariant under gauge transformations depending on arbitrary functions of time (finite degrees of freedom) or space and time (field theory) and the second Noether theorems applies;

ii) the Euler-Lagrange equations cannot be put in normal form, i.e. solved in the accelerations;

iii) the Noether identities implied by the second Noether theorem determine the full set of extra equations of motion to be added to the original Euler-Lagrange equations for consistency;

iv) the solutions of the Euler-Lagrange equations depend on arbitrary functions of time or space and time and, especially in field theory, it may difficult to arrive at a well posed Cauchy problem;

v) there is a number of arbitrary functions of the configuration variables and velocities (primary generalized velocity functions) less or equal to the number of null eigenvalues of the Hessian matrix (assuming that it has constant rank as it happens in many physically relevant systems) determined by the Noether identities;

vi) as a consequence the original configuration variables do not have a predictable evolution in time;

vii) the main task is to identify an equal number of functions of the configuration variables and velocities such that a subset of them has a predictable evolution (*gauge-invariant observables*), while the elements of the complementary subset are left completely arbitrary (*gauge variables*; in general their number is higher than that of the primary generalized velocity functions due to the propagation of their indeterminateness implied by the Noether identities).

Since the Lagrangian formalism does not have a natural technology to face all these problems, we must go to the Hamiltonian formalism, where the theory of canonical transformations allows to find their solution. The degeneracy of the Hessian matrix and the second Noether theorem [13] imply the modification of the Hamiltonian theory known as Dirac-Bergmann theory of constraints [10, 11, 12, 13].

The degeneracy of the Lagrangian implies that the original phase space is restricted to a sub-manifold by as many primary constraints $\phi_A \approx 0$ as null eigenvalues of the Hessian matrix. If H_c is the canonical Hamiltonian, the existence of the constraints leads to the introduction of the Dirac Hamiltonian $H_D = H_c + \sum_A \lambda_A \phi_A$, where the λ_A 's (the *Dirac multipliers*) are arbitrary functions of time or space and time corresponding to generalized velocity functions non-projectable to phase space and left arbitrary by the Euler-Lagrange equations. Then, the so called Dirac algorithm, equivalent to the Noether identities, restricts the number of the arbitrary λ_A 's to that of the primary generalized velocity functions and determines the secondary, tertiary, ... constraints, if any. The full set of constraints determines the final constraint sub-manifold to which the dynamics of the system is restricted. Then the constraints are divided in two groups: i) the *first class* constraints, having weakly zero Poisson brackets with all the constraints and being the generators of the Hamiltonian gauge transformations; ii) the *second class* constraints, which are even in number since they correspond to pairs of redundant canonical variables present, for instance, for reasons of manifest covariance. The final Dirac Hamiltonian is $H_D = H'_c + \sum_a \lambda_a \phi_a$, where the ϕ_a 's are the *primary first class* constraints and the λ_a 's are the Dirac multipliers left arbitrary. Even if there is no general demonstration, in the physically relevant cases H'_c contains all the non-primary first class constraints, each one multiplied by some function of the canonical variables left undetermined by the Hamilton equations. In general H'_c contains also all the second class constraints either linearly or quadratically. When there is reparametrization or diffeomorphism invariance of the action, the canonical Hamiltonian vanishes, $H_c \equiv 0$, and $H'_c \approx 0$.

When there are no second class constraints, as in all the physical system we consider, the constraint manifold is a *presymplectic* manifold and we have the following Poisson bracket algebra: $\{\phi_A, \phi_B\} = C_{AB} \phi_C \approx 0$, $\{H_c, \phi_A\} = C_{AB} \phi_B \approx 0$. The Hamilton-Dirac equations generated by the final Dirac Hamiltonian $H_D = H'_c + \sum_a \lambda_a \phi_a = H''_c + \sum_\alpha f_\alpha \phi_\alpha + \sum_a \lambda_a \phi_a$ (ϕ_α are the non-primary first class constraints, f_α functions on phase space and H''_c is the final canonical Hamiltonian) are equivalent to the Euler-Lagrange equations and to all the extra equations of motion implied by the Noether identities. The *predictable* quantities are the gauge-invariant *Dirac observables* O_r satisfying $\{O_r, \phi_a\} \approx \{O_r, \phi_\alpha\} \approx 0$ and with the deterministic evolution in time ruled by H_c , which is a Dirac observable. The presymplectic constraint sub-manifold is foliated with *gauge orbits*. Each gauge orbit has its points connected by Hamiltonian gauge transformations generated by the first class constraints.

Therefore the gauge orbits are coordinatized by the gauge variables, whose number is equal to the number of first class constraints and which are left arbitrary by the Hamilton-Dirac equations due to a dependence on the arbitrary Dirac multipliers. The reduced phase space, a symplectic manifold coordinatized by the Dirac observables, is the quotient of the presymplectic sub-manifold by the foliation with gauge orbits. Usually it is not a manifold, since rarely we have a nice foliation with all the gauge orbits diffeomorphic. When $H_c \equiv 0$, we get a frozen reduced phase space like in the Hamilton-Jacobi description: the Dirac observables are Jacobi data, an evolution can be reintroduced by using the energy generator of the associated realization of the Poincare' group and it can be shown to be consistent with the gauge motion induced by H_D in the gauge orbits.

The *gauge fixing procedure* is a method to build copies of the reduced phase space by adding as many gauge fixing constraints $\chi_A \approx 0$ as first class constraints $\phi_A \approx 0$. The surface determined by the equations $\chi_A \approx 0$ must intersect each gauge orbit of the presymplectic sub-manifold in one and only one point, namely the gauge variables must be uniquely determined. Locally this is equivalent to the orbit condition $\det |\{\chi_A, \phi_B\}| \neq 0$, but globally it can be a very difficult topological problem to define a good gauge fixing. The orbit condition means that the set of constraints $\phi_A \approx 0$ and $\chi_A \approx 0$ is second class and a symplectic structure isomorphic to that of the reduced phase space is introduced by evaluating the *Dirac brackets* associated to the gauge fixing. The detailed study of the Dirac algorithm and of the Noether identities shows that all the non-primary first class constraints $\phi_\alpha \approx 0$ lie in *chains* whose progenitors are the primary first class constraints $\phi_a \approx 0$ and with the λ_a 's as arbitrary primary generalized velocity functions. This implies the following cascade method as the natural way to introduce gauge fixings. Given for instance a 3-chain $\phi_a \approx 0$ (primary), $\phi_{\alpha_1} \approx 0$ (secondary), $\phi_{\alpha_2} \approx 0$ (tertiary), we first introduce a gauge fixing $\chi_{\alpha_2} \approx 0$ to the tertiary constraint $\phi_{\alpha_2} \approx 0$. Its preservation in time, $\{\chi_{\alpha_2}, H_D\} \approx 0$, generates the gauge fixing $\chi_{\alpha_1} \approx 0$ to the secondary constraint $\phi_{\alpha_1} \approx 0$. Again $\{\chi_{\alpha_1}, H_D\} \approx 0$ generates the gauge fixing $\chi_a \approx 0$ to the primary constraint $\phi_a \approx 0$ and $\{\chi_a, H_D\} \approx 0$ determines the Dirac multiplier λ_a .

See Section II of Ref.[23] for a detailed description, based on Refs. [13, 24], of the theory of singular Lagrangians, of Hamiltonian constraints and of the equivalence of the two approaches.

Let us now consider the main problem, namely the determination of the Dirac observables

and of the gauge variables, or at least of how the original canonical variables depend on them.

A method to find how the original variables depend on the gauge variables makes use of the *multitemporal equations* [25], also called the generalized Lie equations in Ref.[26] for the case in which the constraints satisfy a Poisson bracket Lie algebra. Since the phase space functions f_α in front of the non-primary first class constraints in H_D have an arbitrariness induced by the Dirac multipliers λ_a , we can define an extended Dirac Hamiltonian $H'_D = H_c + \sum_A \lambda_A \phi_A$ where they are replaced by new Dirac multipliers. Let us consider a finite dimensional case with canonical variables $q^i(t)$, $p_i(t)$ and with $\{H_c, \phi_A(q, p)\} = 0$, $\{\phi_A(q, p), \phi_B(q, p)\} = C_{ABC} \phi_C(q, p)$ with C_{ABC} the structure constants of a Lie algebra for the sake of simplicity. Let us define *generalized times* by means of the equations $d\tau_A = \lambda_A(t) dt$ and vector fields $Y_A = A_{AB}(\tau_C) \frac{\partial}{\partial \tau_B}$ satisfying $[Y_A, Y_B] = -C_{ABC} Y_C$. Let us rewrite $q^i(t)$, $p_i(t)$ as $q^i(t, \tau_A)$, $p_i(t, \tau_A)$ and replace the Hamilton equations generated by H'_D with the following coupled multitemporal equations $[F = F(q, p)]$: i) $\frac{\partial F(t, \tau_A)}{\partial t} = \{F, H_c\}$ (namely H_c generates the deterministic t -evolution); ii) $Y_A F(t, \tau_C) = \{F, \phi_A\}$ (these equations generate the gauge orbit through each point of the constraint sub-manifold). The whole set of equations is integrable due to the first-class property of the constraints. In the ideal case in which the gauge foliation is nice, all the gauge orbits are diffeomorphic and in the simplest case all of them are diffeomorphic to the group manifold of a Lie group. In this ideal case to rebuild a gauge orbit from one of its points one needs the Lie equations associated with the given Lie group: the Hamiltonian multitemporal equations are generalized Lie equations describing all the gauge orbits simultaneously. In a generic case this description holds only locally for a set of diffeomorphic orbits, also in the case of systems invariant under diffeomorphisms (in general, when the C_{ABC} are phase space dependent structure functions, the vector fields Y_A have also a dependence upon the canonical variables).

Even if we do not succeed to solve the multitemporal equations, their importance is the connection with the remarkable class of the Shanmugadhasan canonical transformations [24], which allow to separate the gauge variables from the Dirac observables (the tool lacking to the configurational Lagrangian approach). In the finite dimensional case general theorems [27] connected with the Lie theory of function groups [28] ensure the existence of local canonical transformations from the original canonical variables q^i , p_i restricted by the first class constraints (assumed globally defined) $\phi_A(q, p) \approx 0$, to canonical bases P_A , Q_A , P_r , Q_r , such that the equations $P_A \approx 0$ locally define the same original constraint sub-manifold (the P_A are an Abelianization of the first class constraints); the Q_A are the adapted Abelian

gauge variables describing the gauge orbits (they are a realization of the generalized times, $\tau_A = Q_A$ of the multitemporal equations); the Q_r, P_r are an adapted canonical basis of Dirac observables. These canonical transformations are the basis of the Hamiltonian definition of the Faddeev-Popov measure of the path integral [29] and give a trivialization of the BRS construction of observables (the BRS method works when the first class constraints may be Abelianized [12]). Second class constraints, when present, are also taken into account by the Shanmugadhasan canonical transformation [23]: they correspond to pairs of weakly vanishing irrelevant canonical variables.

Therefore the problem of the search of the Dirac observables becomes the problem of finding Shanmugadhasan canonical transformations. Often, especially in field theories, it is not known how to express the gauge variables and the Dirac observables in terms of the original canonical variables. But the study of the finite Hamiltonian gauge transformations, namely of the multitemporal equations, usually allows to find the inverse canonical transformation (or at least its restriction to the constraint sub-manifold) $q^i = q^i[Q_A, Q_r, P_r]$, $p_i = p_i[Q_A, Q_r, P_r]$. This is enough to define the preferred gauge fixings $Q_A \approx 0$, in which all the physical quantities can be explicitly expressed in terms of the predictable Dirac observables. If a system with constraints admits one (or more) global Shanmugadhasan canonical transformations, one obtains one (or more) privileged global gauges in which the physical Dirac observables are globally defined and globally separated from the gauge degrees of freedom. These privileged gauges (when they exist) can be called *generalized Coulomb gauges*. When the system under investigation has some global symmetry group, the associated theory of the momentum map is a source of globality.

Let us add some remarks [23] on the properties of the physical gauge systems defined on flat Minkowski space-time deriving from the existence of the *global Poincare' symmetry*. In this case we must study the structure of the constraint sub-manifold from the point of view of the orbits of the Poincare' group. If p^μ is the total momentum of the system, the constraint manifold has to be divided in four strata (some of them may be absent for certain systems) according to whether $\epsilon p^2 > 0$, $p^2 = 0$, $\epsilon p^2 < 0$ or $p^\mu = 0$ [the metric is $\epsilon (+ - - -)$ with $\epsilon = \pm 1$ according to the particle physics or general relativity convention]. Due to the different little groups of the various Poincare' orbits, the gauge orbits of different sectors will not be diffeomorphic. Therefore the constraint sub-manifold is a stratified manifold and the gauge foliations of relativistic systems are nearly never nice, but rather one has to do with singular foliations. For an acceptable relativistic system the stratum $\epsilon p^2 < 0$ has

to be absent to avoid tachyons. To study the strata $p^2 = 0$ and $p^\mu = 0$ one has to add these relations as extra constraints. For all the strata the next step is to do a canonical transformation from the original variables to a new set consisting of center-of-mass variables x^μ , p^μ and of variables relative to the center of mass. Let us now consider the stratum $\epsilon p^2 > 0$. By using the standard Wigner boost $L_\nu^\mu(p, \overset{\circ}{p})$ ($p^\mu = L_\nu^\mu(p, \overset{\circ}{p}) \overset{\circ}{p}^\nu$, $\overset{\circ}{p}^\mu = \eta \sqrt{\epsilon p^2} (1; \vec{0})$, $\eta = \text{sign } p^0$), one boosts the relative variables at rest. The new variables are still canonical and the base is completed by p^μ and by a new center-of-mass coordinate \tilde{x}^μ , differing from x^μ for spin terms. The variable \tilde{x}^μ has complicated covariance properties; instead the new relative variables are either Poincaré' scalars or Wigner spin-1 vectors, transforming under the group $O(3)(p)$ of the Wigner rotations induced by the Lorentz transformations. A final canonical transformation [30], leaving fixed the relative variables, sends the center-of-mass coordinates \tilde{x}^μ , p^μ in the new set $p \cdot \tilde{x} / \eta \sqrt{\epsilon p^2} = p \cdot x / \eta \sqrt{\epsilon p^2}$ (the time in the rest frame), $\eta \sqrt{\epsilon p^2}$ (the total mass), $\vec{k} = \vec{p} / \eta \sqrt{\epsilon p^2}$ (the spatial components of the 4-velocity $k^\mu = p^\mu / \eta \sqrt{\epsilon p^2}$, $k^2 = 1$), $\vec{z} = \eta \sqrt{\epsilon p^2} (\vec{\tilde{x}} - \tilde{x}^0 \vec{p} / p^0)$. \vec{z} is a non-covariant center-of-mass canonical 3-coordinate multiplied by the total mass: it is the classical analog of the Newton-Wigner position operator (like it, \vec{z} is covariant only under the little group $O(3)(p)$ of the time-like Poincaré orbits). This techniques are useful to find Lorentz scalar Abelianizations of the first class constraints and shows that the breaking of manifest Lorentz covariance is restricted to the decoupled, physically irrelevant, center-of-mass motion.

In gauge field theories the situation is more complicated, because the theorems ensuring the existence of the Shanmugadhasan canonical transformation have not been extended to the infinite-dimensional case. One of the reasons is that some of the constraints can now be interpreted as elliptic equations and they can have *zero modes*. Let us consider the stratum $\epsilon p^2 > 0$ of free Yang-Mills theory as a prototype and its first class constraints, given by the Gauss laws and by the vanishing of the time components of the canonical momenta. The problem of the zero modes will appear as a singularity structure of the gauge foliation of the allowed strata, in particular of the stratum $\epsilon p^2 > 0$. This phenomenon was discovered in Ref.[31] by studying the space of solutions of Yang-Mills and Einstein equations, which can be mapped onto the constraint manifold of these theories in their Hamiltonian description. It turns out that the space of solutions has a *cone over cone* structure of singularities: if we have a line of solutions with a certain number of gauge symmetries, in each point of this line there is a cone of solutions with one less symmetry. In the Yang-Mills case the *gauge symmetries* of a gauge potential are connected with the generators of its stability

group, i.e. with the subgroup of those special gauge transformations which leave invariant that gauge potential (this is the *Gribov ambiguity* for gauge potentials; there is also a more general Gribov ambiguity for field strengths, the *gauge copies* problem; see Refs.[32] for a review). The analog of gauge symmetries in general relativity is the existence of *Killing vectors* implying that the space-time has symmetries.

Since the Gauss laws are generators of Hamiltonian gauge transformations (and depend on the chosen gauge potential through the covariant derivative), this means that for a gauge potential with non trivial stability group those combinations of the Gauss laws corresponding to the generators of the stability group cannot be any more first class constraints, since they do not generate effective gauge transformations but special symmetry transformations. This problematic has still to be clarified, but it seems that in this case these components of the Gauss laws become *third class* constraints, which are not generators of true gauge transformations. This new kind of constraints was introduced in Refs.[13, 24] in the finite dimensional case as a result of the study of some examples, in which the Jacobi equations (the linearization of the Euler-Lagrange equations) are singular, i.e. some of their solutions are not infinitesimal deviations between two neighboring extremals of the Euler-Lagrange equations. This interpretation seems to be confirmed by the fact that the singularity structure discovered in Ref.[31] follows from the existence of singularities of the linearized Yang-Mills and Einstein equations. These problems are part of the Gribov ambiguity, which, as a consequence, induces an extremely complicated stratification and also singularities in each Poincaré stratum of the constraint sub-manifold.

Other possible sources of (not yet explored) singularities of the gauge foliation of Yang-Mills theory in the stratum $\epsilon p^2 > 0$ may be: i) different classes of gauge potentials identified by different values of the field invariants; ii) the orbit structure of the rest frame (or Thomas) spin \vec{S} , identified by the Pauli-Lubanski Casimir $W^2 = -\epsilon p^2 \vec{S}^2$ of the Poincaré' group.

The final outcome of this structure of singularities is that the reduced phase-space, i.e. the space of the gauge orbits, is in general a stratified manifold with singularities [33]. In the stratum $\epsilon p^2 > 0$ of the Yang-Mills theory these singularities survive the Wick rotation to the Euclidean formulation and it is not clear how the ordinary path integral approach and the associated BRS method can take them into account. The search of a global canonical basis of Dirac observables for each stratum of the space of the gauge orbits can give a definition of the measure of the phase space path integral, but at the price of a non polynomial Hamiltonian. Therefore, if it is not possible to eliminate the Gribov ambiguity (assuming

that it is only a mathematical obstruction without any hidden physics), the existence of global Dirac observables for Yang-Mills theory is very problematic.

See Ref.[23] for the list of special relativistic systems, from relativistic particle mechanics to the Nambu string and the $SU(3) \times SU(2) \times U(1)$ standard model of elementary particles [34], whose Dirac observables and Abelianized gauge variables have been determined by means of Shanmugadhasan canonical transformations.

III. SIMULTANEITY AND PARAMETRIZED MINKOWSKI THEORIES.

In this Section we explore those aspects of special relativistic systems which suitably modified are present also in general relativity, whose formulation must be made in a way allowing a deparametrization to special relativity.

A. The Lesson of Relativistic Mechanics.

Relativistic particle mechanics in presence of interactions with a finite time delay goes back [14] to the Tetrode-Fokker action principle, to the Feynman-Wheeler electrodynamics and to its generalization by Van Dam and Wigner. The particle world-lines $q_i^\mu(\tau^i)$, $i = 1, \dots, N$, are parametrized with independent affine parameters τ^i and these action principles are invariant under separate reparametrizations of each world-line, since this is geometrically possible even in presence of interactions. Since the dynamical correlation among the points on the particle's world-lines is not in general one-to-one in these approaches, it was impossible to develop a Hamiltonian formulation starting from the Euler-Lagrange integro-differential equations of motion implied by the delay. The natural development of these approaches was field theory, for instance the study of the coupled system of relativistic charged particles plus the electro-magnetic field.

These difficulties and the need of a description of relativistic bound states led to the development of relativistic mechanics with action-at-a-distance interactions described by suitable potentials implying a one-to-one correlation among the world-lines. As already said, geometrically each particle has its world-line described by a four-vector (the *four-position*) $q_i^\mu = q_i^\mu(\tau^i)$, $i = 1, \dots, N$, parametrized with an independent arbitrary affine scalar parameter τ^i ¹. By inverting $q_i^o(\tau^i)$ to get $\tau^i = \tau^i(q_i^o)$, we can identify the world-line in a non-manifestly covariant way with $\vec{q}_i = \vec{q}_i(q_i^o)$: in this form they are named *predictive coordinates*. The instant form amounts to put $q_1^o = \dots = q_N^o = x^o$ and to describe the world-lines with the functions $\vec{q}_i(x^o)$. Each one of these configuration variables has a different associated notion of *velocity*: $\frac{dq_i^\mu(\tau^i)}{d\tau^i}$ (or $\frac{dq_i^\mu(\tau)}{d\tau}$), $\frac{d\vec{q}_i(q_i^o)}{dq_i^o}$ (*predictive velocities*), $\frac{d\vec{q}_i(x^o)}{dx^o}$, and of *acceleration*: $\frac{d^2 q_i^\mu(\tau^i)}{(d\tau^i)^2}$ (or $\frac{d^2 q_i^\mu(\tau)}{d\tau^2}$), $\frac{d^2 \vec{q}_i(q_i^o)}{(dq_i^o)^2}$ (*predictive accelerations*), $\frac{d^2 \vec{q}_i(x^o)}{(dx^o)^2}$.

Bel's *non-manifestly covariant predictive mechanics* [35] is the attempt

¹ The standard choice in the manifestly covariant approach with a $4N$ -dimensional configuration space is $\tau^1 = \dots = \tau^N = \tau$. Another possibility is the choice of proper times $\tau^i = \tau_{PT}^i$.

to describe relativistic mechanics with N -time predictive equations of motion for the predictive coordinates $\vec{q}_i(q_i^o)$ in Newtonian form: $m_i \frac{d^2 \vec{q}_i(q_i^o)}{(dq_i^o)^2} \stackrel{\circ}{=} \vec{\mathcal{F}}_i(q_k^o, \vec{q}_k(q_k^o), \frac{d\vec{q}_k(q_k^o)}{dq_k^o})$. Since the left hand side of these equations depends only on q_i^o , the *predictive forces* must satisfy the *predictive conditions* $\frac{d\vec{\mathcal{F}}_i}{dq_k^o} = 0$ for $k \neq i$. Moreover they must be invariant under space translations and behave like space three-vectors under spatial rotations. Finally they must satisfy the Currie-Hill equations [36] (or *Currie-Hill world-line conditions*), whose satisfaction implies that the predictive positions $\vec{q}_i(x^o)$ behave under Lorentz boosts like the spatial components of four-vectors. Bel [37] proved that these equations constitute the necessary and sufficient conditions, which guarantee that the dynamics is Lorentz invariant with respect to finite Lorentz transformations. However the Currie-Hill equations are so non-linear that it is practically impossible to find consistent predictive forces and develop this point of view.

The first well posed Hamiltonian formulation of relativistic mechanics was given by Dirac [38] with the *instant, front (or light) and point forms* of relativistic Hamiltonian dynamics and the associated canonical realizations of the Poincare' algebra. In the instant form the simultaneity hyper-surfaces defining a parameter for the time evolution are space-like hyper-planes $x^o = \text{const.}$, in the front form hyper-planes $x^- = \frac{1}{2}(x^o - x^3) = \text{const.}$ tangent to future light-cones, while in the point form the future branch of a two-sheeted hyperboloid $x^2 > 0$. In a $6N$ -dimensional phase space for N scalar particles the ten generators of the Poincare' algebra are classified into *kinematical* generators (the generators of the stability group of the simultaneity hyper-surface) and *dynamical* generators (the only ones to be modified with respect to the free case in presence of interactions) according to the chosen concept of simultaneity. While in the instant and point forms there are four dynamical generators (in the former energy and boosts, in the latter the four-momentum), the front form has only three of them. After the pioneering work of Thomas, Bakamjian and Foldy [39] on the non-manifestly covariant Hamiltonian instant form, much work has been done in elucidating the classical and quantum aspects of this approach, which has a well defined non-relativistic limit and $1/c$ expansions containing the deviations (potentials) from the free case.

A big obstacle for the development of Hamiltonian models was the *no-interaction theorem* of Currie, Jordan and Sudarshan [40] (see Refs.[41] for reviews). Its original form was formulated in the Hamiltonian Dirac instant form in the $6N$ -dimensional phase space $(\vec{q}_i(x^o), \vec{p}_i(x^o))$ of N particles. The no-interaction theorem states that the hypotheses i) the

configuration variables $\vec{q}_i(x^o)$ are canonical, i.e. $\{\vec{q}_i(x^o), \vec{q}_j(x^o)\} = 0$; ii) the Lorentz boosts can be implemented as canonical transformations (existence of a canonical realization of the Poincaré group) and the $\vec{q}_i(x^o)$ are the space components of four-vectors; iii) the system is non-singular (the transformation from positions and velocity to canonical coordinates is non singular; it is not assumed the existence of a Lagrangian); imply *only free motion*.

As a consequence of the theorem, if we denote $x_i^\mu(\tau)$, $p_{i\mu}(\tau)$ the canonical coordinates of the manifestly covariant approach and $\vec{x}_i(x^o)$, $\vec{p}_i(x^o)$ their equal time restriction in the instant form, we have $\vec{x}_i(x^o) \neq \vec{q}_i(x^o)$ except for free motion. Let us remark that, *since the manifestly covariant approach gives the classical basis for the theory of covariant wave equations, the four-coordinates $x_i^\mu(\tau)$ (and not the geometrical four-positions $q_i^\mu(\tau)$) are the coordinates locally minimally coupled to external fields.*

Many attempts were made to avoid this theorem by relaxing one of its hypotheses or by renouncing to the concept of world-line. One was the already quoted *non-manifestly covariant manifestly predictive approach*. Then the *manifestly covariant, non-manifestly predictive approach* was developed [42]: in it the world-lines are described by N four-vectors $q_i^\mu(\tau_{PT}^i)$ parametrized with N proper times τ_{PT}^i : $\left(\frac{dq_i^\mu(\tau_{PT}^i)}{d\tau_{PT}^i}\right)^2 = m_i^2$. The covariant equations of motion are $\frac{d^2 q_i^\mu(\tau_{PT}^i)}{(d\tau_{PT}^i)^2} \stackrel{\circ}{=} \theta_i^\mu(q_k^\nu(\tau_{PT}^k), \frac{dq_k^\mu(\tau_{PT}^k)}{d\tau_{PT}^k})$ with $\theta_i^\mu \frac{dq_i^\mu}{d\tau_{PT}^i} = 0$. In this many-time formalism the predictive conditions are equivalent to the existence of N Abelian, Poincaré invariant vector fields (identified for the first time by Droz Vincent) \mathcal{H}_i , $[\mathcal{H}_i, \mathcal{H}_j] = 0$, such that $\mathcal{H}_i q_j^\mu(\tau_{PT}^j) = \delta_{ij} \frac{dq_i^\mu(\tau_{PT}^i)}{d\tau_{PT}^i}$, $\mathcal{H}_i \frac{dq_j^\mu(\tau_{PT}^j)}{d\tau_{PT}^j} = \delta_{ij} \theta_i^\mu$. The connection with the non-manifestly covariant manifestly predictive approach is obtained by imposing $q_1^o(\tau_{PT}^1) = \dots = q_N^o(\tau_{PT}^N) = x^o$ and by means of the identification $\vec{q}_i(\tau_{PT}^i) = \vec{q}_i(x^o)$. For the predictive forces one gets $\mathcal{F}_i^h = \frac{1}{m_i^2} \left(1 - \left(\frac{d\vec{q}_i(x^o)}{dx^o}\right)^2\right) \left(\delta^{hk} - \frac{dq_i^h(x^o)}{dx^o} \frac{dq_i^k(x^o)}{dx^o}\right) \theta_i^k$ with the Currie-Hill conditions satisfied.

Finally independently Droz Vincent's many-time Hamiltonian formalism [43] (a refinement of the manifestly covariant non-manifestly predictive approach; it is the origin of the multi-temporal equations quoted in the previous Section), Todorov's quasi-potential approach to bound states [44] and Komar's study of toy models for general relativity [45] *converged towards manifestly covariant models based on singular Lagrangians and/or Dirac-Bergmann theory of constraints*. Since quite often the Lagrangian formulation is not known, a system of N relativistic scalar particles is usually described in a manifestly covariant $8N$ -dimensional phase space with coordinates $(x_i^\mu(\tau), p_{i\mu}(\tau))$ $[\{x_i^\mu(\tau), p_{j\nu}(\tau)\} = -\delta_{ij} \delta_\nu^\mu, \{x_i^\mu(\tau), x_j^\nu(\tau)\} = \{p_{i\mu}(\tau), p_{j\nu}(\tau)\} = 0]$, where τ is a scalar evolution parameter. The description is independent from the choice of τ : the Lagrangian (even if usually not explic-

itly known) is τ -reparametrization invariant, while at the Hamiltonian level the canonical Hamiltonian vanishes identically, $\bar{H}_c \equiv 0$. Since the physical degrees of freedom for N scalar particles are $6N$, there are constraints, which, in the case of N free scalar particles of mass m_i , are just the mass-shell conditions

$$\begin{aligned}\bar{\phi}_i(q, p) &= p_i^2 - m_i^2 \approx 0, \quad i = 1, \dots, N, \quad \Rightarrow \quad p_i^0 \approx \pm \sqrt{m_i^2 + \vec{p}_i^2}, \\ \{\bar{\phi}_i(q, p), \bar{\phi}_j(q, p)\} &= 0.\end{aligned}\tag{3.1}$$

These constraints say that the *time* variables $x_i^0(\tau)$ are the *gauge variables of a τ -reparametrization invariant theory with $\bar{H}_c \equiv 0$* . The Dirac Hamiltonian is $\bar{H}_D = \sum_{i=1}^N \lambda^i(\tau) \bar{\phi}_i$ if all the first class constraints are primary. In the free case this is true because these constraints are implied by the action principle ²

$$\begin{aligned}S &= \int d\tau L, \quad L = - \sum_{i=1}^N m_i \sqrt{\dot{x}_i^2(\tau)}, \\ \Rightarrow \quad p_{i\mu} &= - \frac{\partial L}{\partial \dot{x}_i^\mu} = m_i \frac{\dot{x}_i^\mu(\tau)}{\sqrt{\dot{x}_i^2(\tau)}}, \quad p_i^2 - m_i^2 \approx 0.\end{aligned}\tag{3.2}$$

The Euler-Lagrange equations are $L_{i\mu} = \frac{d}{d\tau} \frac{m_i \dot{x}_i^\mu(\tau)}{\sqrt{\dot{x}_i^2(\tau)}} \stackrel{\circ}{=} 0$, $i = 1, \dots, N$, while the Hamilton-Dirac equations are $\dot{x}_i^\mu(\tau) \stackrel{\circ}{=} -2\lambda^i(\tau) p_i^\mu$, $\dot{p}_{i\mu} \stackrel{\circ}{=} 0$, $p_i^2 - m_i^2 \approx 0$. The final constraint sub-manifold is the union of 2^N (for generic masses m_i) disjoint sub-manifolds corresponding the choice of the either positive- or negative-energy branch of each two-sheeted mass-shell hyperboloid. Each branch is a non-compact sub-manifold of phase space on which each particle has a well defined sign of the energy and 2^N is a topological number (the zeroth homotopy class of the constraint sub-manifold) ³.

Only in the case of two-body systems it is known how to introduce interactions due to the Droz-Vincent-Todorov-Komar model [43, 44, 45] with an *arbitrary action-at-a-distance*

² An alternative N-time Hamiltonian description is the multi-temporal one of the previous Section, in which the N scalar time parameters τ^i are defined by $d\tau^i = \lambda^i(\tau) d\tau$.

³ Let us remark that when the particles are coupled to weak external fields the 2^N sub-manifolds are deformed but remain disjoint. But when the strength of the external fields increases the various sub-manifolds may intersect each other and this topological discontinuity is the signal that we are entering in a non-classical regime where quantum pair production becomes relevant due to the disappearance of mass gaps.

interaction instantaneous in the rest frame described by the two first class constraints $\bar{\phi}_i = p_i^2 - m_i^2 + V(r_\perp^2) \approx 0$, $i=1,2$, with $r_\perp^\mu = (\eta^{\mu\nu} - p^\mu p^\nu / \epsilon p^2) r_\nu$, $r^\mu = x_1^\mu - x_2^\mu$, $p_\mu = p_{1\mu} + p_{2\mu}$. For $N > 2$ a closed form of the N first class constraints is not known explicitly (there is only an existence proof): only versions of the model with explicit gauge fixings, so that all the constraints except one are second class, are known.

This model has been completely understood both at the classical and quantum level [30]. Its study led to the identification of a class of canonical transformations (utilizing the standard Wigner boost for time-like Poincaré orbits) which allowed to understand how to define suitable center-of-mass and relative variables (in particular a suitable relative energy is determined by a combination of the two first class constraints, so that the relative time variable is a gauge variable), how to find a quasi-Shanmugadhasan canonical transformation adapted to the constraint determining the relative energy, how to separate the four, topologically disjoint, branches of the mass spectrum (it is determined by the other independent combination of the constraints; therefore, there is a distinct Shanmugadhasan canonical transformation for each branch). At the quantum level it was possible to find four physical scalar products, compatible with both the resulting coupled wave equations (i.e. independent from the relative and the absolute rest-frame times): they have been found as generalization of the two existing scalar products of the Klein-Gordon equation: all of them are non-local even in the limiting free case and differ among themselves for the sign of the norm of states on different mass-branches. This example shows that the physical scalar product knows the functional form of the constraints.

The no-interaction theorem is initially avoided due to the singular nature of the Lagrangian: there is a canonical realization of the Poincaré' group and the canonical coordinates x_i^μ are four-vectors. However, when we restrict ourselves to the constraint sub-manifold and look for canonical coordinates adapted to it and to the Poincaré' group, it turns out that among the final canonical coordinates will always appear the canonical *non-covariant* center of mass \tilde{x}^μ of the particle system. Therefore, all these models have the following properties: i) the canonical and predictive four-positions do not coincide (except in the free case); ii) the decoupled canonical center of mass is not covariant.

These models with N first class constraints have the following interpretation. Since there are N arbitrary Dirac multipliers $\lambda^i(\tau)$ [the Dirac Hamiltonian is $H_D = \sum_{i=1}^N \lambda^i(\tau) \phi_i(q, p)$], the solutions of the Euler-Lagrange equations are $x_i^\mu(\tau) \stackrel{\circ}{=} x_i^\mu[\lambda^1(\tau), \dots, \lambda^N(\tau)] \stackrel{def}{=} x_i^\mu(\tau^1, \dots, \tau^N) \neq$

$q_i^\mu(\tau^i)$. Only for free particles we get $x_i^\mu(\tau) \stackrel{\circ}{=} x_i^\mu[\lambda^i(\tau)] \stackrel{def}{=} x_i^\mu(\tau^i) = q_i^\mu(\tau^i)$. Therefore *in the interacting case the canonical coordinates cannot coincide with the predictive ones except in the free case* as required by the no-interaction theorem. Each *gauge-dependent* canonical coordinate x_i^μ spans a N -dimensional hypersurface (parametrized by the multi-times τ^1, \dots, τ^N), the Hamiltonian world-sheet, instead of a world-line (parametrized by τ^i). Only if we make $N - 1$ pre-gauge fixings $\lambda^i(\tau) = \Lambda^i(\lambda(\tau))$ with given functions Λ^i of only one arbitrary function $\tilde{\tau} = \lambda(\tau)$, we can select *a well defined world-line* $\hat{x}_i^\mu(\tilde{\tau})$ (parametrized by $\tilde{\tau} = \lambda(\tau)$) for each particle: *it will be interpreted as the predictive world-line* $\hat{q}_i^\mu(\tilde{\tau})$ *of that gauge*. The $N - 1$ pre-gauge fixings can be replaced by $N - 1$ real gauge fixing constraints (implying them) $\bar{\chi}_a(q, p) \approx 0$, $a = 1, \dots, N - 1$, which are interpreted as a statement on the $N - 1$ gauge variables *relative times* ($N - 1$ independent combinations of the variables $x_i^0(\tau) - x_j^0(\tau)$). But this is *equivalent to a choice of which one-to-one space-like correlation among the particles we are going to use to describe the system of N interacting particles* and, therefore, of which kind of triggering of the particles the inertial observer in the laboratory is going to use to detect them. Since a change of Hamiltonian gauge is equivalent to a (in general) *non-point* canonical transformation in phase space, corresponding to a Lie-Backlund transformation on the velocity space, *to each gauge is associated a different configuration space* (to be reached by inverse Legendre transformation). All these configuration spaces can be identified with different copies of Minkowski space-time containing different set of world-lines connected by the velocity-dependent Lie-Backlund transformations. In this way we reproduce all the possible world-lines spanning the world-sheet in the Hamiltonian description. We need a *semantic* statement [45] about which is the configuration space which contains those specific world-lines which are more natural from a physical interpretational viewpoint. This choice is equivalent to select a *natural* (i.e. preferred for the physical interpretation) set of gauge fixings and this, in turn, selects a certain one-to-one space-like correlation to be associated to the given type of interaction. The chosen world-lines in Minkowski space-time will be the *natural predictive world-lines* according to the chosen interpretation. The natural set of gauge fixings for nearly all the models till now proposed is to choose the one-to-one space-like correlation corresponding to *simultaneity in the (inertial) rest frame of the isolated N -body system*: the $N - 1$ natural gauge fixing constraints are $N - 1$ independent combinations of $p_\mu [x_i^\mu(\tau) - x_j^\mu(\tau)] \approx 0$, where p^μ is the conserved total four-momentum of the isolated N -body system.

To clarify the interpretation we need a quasi-Shanmugadhasan canonical transformation adapted to those $N - 1$ combinations of the N first class constraints, whose $N - 1$ gauge fixings select the natural world-lines for the given interaction (usually those whose points are in one-to-one correlation in the rest frame). In the $N = 2$ case (DVTK model) it is given by [30]

$$\begin{bmatrix} x_i^\mu \\ p_i^\mu \end{bmatrix} \longrightarrow \begin{bmatrix} T & T_R \\ \epsilon & \hat{\epsilon}_R (\approx 0) \end{bmatrix} \begin{bmatrix} \vec{z} & \vec{\rho} \\ \vec{k} & \vec{\pi} \end{bmatrix} \quad (3.3)$$

where $T_R = p \cdot r / \epsilon$, $\epsilon = \sqrt{p^2}$, $T = p \cdot x / \epsilon$ [$x^\mu = \frac{1}{2}(x_1^\mu + x_2^\mu)$], $\epsilon_R = \frac{1}{2\epsilon}(\bar{\phi}_1 - \bar{\phi}_2) = \frac{1}{2}[p \cdot q - \frac{1}{2}(m_1^2 - m_2^2)] \approx 0$ [$q^\mu = \frac{1}{2}(p_1^\mu - p_2^\mu)$]. While the 3-center-of-mass quantities \vec{z} and \vec{k} have been defined at the end of the previous Section, the relative 3-vectors $\rho^r = \epsilon_\mu^r(p) r^\mu$, $\pi^r = \epsilon_\mu^r(p) q^\mu$ require the columns of the standard Wigner boost for time-like Poincare' orbits for their definition. The gauge variable conjugate to the constraint $\epsilon_R \approx 0$ is the Lorentz-scalar relative time T_R . As a consequence the natural gauge fixing, identifying the natural world-lines of this model inside the Hamiltonian world-sheet, is $\bar{\chi}_- = T_R \approx 0$, i.e. instantaneous interaction in the rest frame. A different gauge fixing would identify a different pair of world-lines in the world-sheet: its being non-natural is also shown by the Shanmugadhasan canonical transformation, which takes into account the fact that the potential depends upon r_\perp^μ , the mutual separation in the rest frame.

The other combination $\bar{\phi}_+ = \frac{1}{2}(\bar{\phi}_1 + \bar{\phi}_2) \approx 0$ of the two first class constraints is an equation for the mass spectrum ϵ of the system. The natural gauge fixing to it, $\bar{\chi}_+ = T - \tau \approx 0$, is a choice of the Lorentz-scalar rest-frame time T as the parameter to be used for the overall evolution.

For N particles there is a combination $\bar{\phi}_+ \approx 0$ of the N first class constraints determining the mass spectrum and generating τ -reparametrizations of the overall isolated system, and $N - 1$ combinations $\bar{\phi}_a \approx 0$, $a = 1, \dots, N - 1$, which do not generate reparametrizations but Hamiltonian gauge transformations implying the gauge nature of $N - 1$ independent relative times, to be fixed with $N - 1$ natural gauge fixings $\bar{\chi}_a \approx 0$.

Let us remark, that, as shown in Ref. [46], once we have done the semantic choice natural for the given interactions, a set of N gauge fixing $\bar{\chi}_i \approx 0$ is *admissible* if: i) they imply that the $N - 1$ natural gauge fixings $\bar{\chi}_a \approx 0$, identifying the natural world-lines in the Hamiltonian world-sheet, allow to express all the Dirac multipliers $\lambda_i(\tau)$ as Poincare-invariant functions of a unique arbitrary multiplier $\lambda(\tau)$; ii) they imply a final gauge fixing $\bar{\chi}_+ \approx 0$ which

is Lorentz invariant (otherwise, like it happens with $x^o(\tau) \approx \dots \approx x_N^o(\tau) \approx \tau$, the selected world-lines are not stable under Poincare' transformations and the whole Hamiltonian world-sheet reappears).

Since we have $\{x_i^\mu, \bar{\phi}_j\} \neq 0$ for $i \neq j$ (the constraints do not generate reparametrizations of a single world-line) except in the free case, in general, as already said, the canonical coordinates $x_i^\mu(\tau)$ do not coincide with any set of predictive ones $q_i^\mu(\tau^i)$ (associated to the various world-lines existing in the Hamiltonian world-sheet), which, if expressed in terms of x_i^μ, p_i^μ , should satisfy $\{q_i^\mu, \bar{\phi}_j\} = 0$ for $i \neq j$. However, if we add a set of admissible gauge fixings $\bar{\chi}_i \approx 0$, we select well defined world-lines for which we must have geometrically $x_i^\mu|_{\bar{\chi}_j=0} \approx q_i^\mu$. However, if we go to the Dirac brackets implied by $\bar{\phi}_i \approx 0, \bar{\chi}_i \approx 0$, we find that in the $6N$ -dimensional reduced phase space we get $x_i^\mu(\tau) \equiv q_i^\mu(\tau^i(\tau))$ but with these quantities not being any more four-vectors in accord with the no-interaction theorem. Actually the Dirac brackets force us to make Poincare' transformations with a fixed parameter $\tau \equiv T$ and this violates the Hamiltonian world-line condition and also the predictive Currie-Hill one: i) in the Hamiltonian approach to implement manifest Poincare' covariance we need to add to each Poincare' transformations (essentially to the boosts) a compensating τ -reparametrization which is forbidden in the reduced phase space; ii) to implement the individual τ^i -reparametrization of each world-line we have to use the predictive accelerations which correlate world-line points simultaneous in a generic (in general non inertial) frame and not only in the inertial rest frame as implied by the Dirac brackets.

The lesson of relativistic mechanics is the need of i) a good choice of simultaneity adapted to the type of action-at-a-distance interaction under investigation; ii) a set of relativistic center-of-mass and relative canonical variables compatible with this choice ($p \cdot r_a \approx 0$). Given a set of first class constraints the main tool for solving these problems are the Shanmugadhasan canonical transformations adapted to as many constraints as possible.

This state of affairs becomes a necessity when we consider a charged scalar particle interacting with the electro-magnetic field. Starting from the traditional action principle $S = \int d\tau \left[-m \sqrt{\epsilon \dot{x}^2(\tau)} + e \dot{x}^\mu(\tau) A_\mu(x(\tau)) \right] + \frac{1}{2} \int d^4z F^{\mu\nu}(z^o, \vec{z}) F_{\mu\nu}(z^o, \vec{z})$, we arrive at the primary constraints $[p_\mu - e A_\mu(x(\tau))]^2 - m^2 \approx 0, \Pi^o(z^o, \vec{z}) \approx 0$ in a phase space spanned by $x^\mu(\tau), p_\mu(\tau), A_\mu(z^o, \vec{z}), \Pi^\mu(z^o, \vec{z})$. Since there is no concept of *equal time* valid both for the particle and the field, we do not know how to define the Poisson bracket of these two constraints.

To solve this problem without explicitly breaking manifest Lorentz covariance [for instance by imposing by hand $x^o(\tau) = z^o$ as a restriction on the affine parameter τ], we need to revisit the notion of simultaneity in special relativity and to introduce the 3+1 point of view which is implied by the Tomonaga-Schwinger approach to quantum field theory [47] and is a prerequisite to move to globally hyperbolic pseudo-Riemannian manifolds as it is required by the Hamiltonian formulation of metric and tetrad gravity.

B. Simultaneity Notions in Special Relativity.

Let us review some recent results [22] induced by a revisitation of the problem of simultaneity in special relativity.

In absence of gravity the special relativistic description of physical systems is done in Minkowski space-time M^4 , a flat pseudo-Riemannian 4-manifold with Lorentz signature. The *relativity principle* states that the laws of physics are the same in a special family of *rigid systems of reference*, the *inertial systems*, in uniform translational motion one with respect to the other, endowed with pseudo-Cartesian (Lorentzian) 4-coordinates where the metric has the form $\epsilon(+ - - -)$. In them the laws of physics are manifestly covariant under the kinematical group of Poincaré' transformations (constant translations and Lorentz transformations) and velocity has no absolute meaning.

An *observer* is a time-like future oriented world-line γ in M^4 ; in Cartesian coordinates we have $\gamma : R \mapsto M^4$, $\tau \mapsto x^\mu(\tau)$, $\epsilon \dot{x}^2(\tau) > 0$ [$\dot{x}^\mu(\tau) = dx^\mu(\tau)/d\tau$]. An *inertial observer* has a constant 4-velocity $\dot{x}^\mu(\tau) = \text{const.}$, i.e. the world-line is a straight line. An *instantaneous inertial observer* is any point P on γ together with the unit time-like vector $e_{(o)}^\mu = \dot{x}^\mu / \sqrt{\epsilon \dot{x}^2}$ tangent to γ at P . An *inertial system* I_P with origin at P has γ as time axis and three orthogonal space-like straight lines orthogonal to γ in P , with unit tangent vectors $e_{(r)}^\mu$, $r = 1, 2, 3$, as space axes. It corresponds to a congruence of inertial observers defined by the constant unit vector field $e_{(o)}^\mu$. The four orthonormal vectors $e_{(\alpha)}^\mu$ are a tetrad at P and each inertial observer of the congruence is endowed with a standard atomic clock ⁴. A *reference*

⁴ Usually, in the case of an isolated inertial observer, the clock is assumed to measure the proper time of the observer. However, let us remark that a priori the notion of proper time requires the solution of the equations of motion generating the observer world-line, something which is beyond the duties of an experimentalist. As a consequence, the standard unit of time is a *coordinate second* [48] and not a proper time both in special and general relativity. In the case of a congruence of observers (the points of an extended laboratory) this notion of coordinate time emphasizes the conventional nature of the

frame (or *system of reference* or *platform*) is a congruence of time-like world-lines, namely a unit vector field $u^\mu(x)$ having these world-lines as integral curves.

The basic problem of relativity is the *absence of an absolute notion of simultaneity*, so that the synchronization of distant clocks, the definition of an instantaneous 3-space, the spatial distance between events at space-like separation and the one-way velocity of light are *frame-dependent* concepts. Regarding light there are *two independent postulates* in special relativity, which state that the *round-trip* (or *two-way*)⁵ velocity of light is the same, c , in every inertial system (the round-trip postulate) and *isotropic* (the light postulate). Instead the *one-way* velocity of light between two events depends on the definition of synchronization of the two clocks on the two world-lines containing the given events and may be i) non-isotropic, ii) lesser or higher than c [49]. The standard theory of measurements in special relativity is defined in inertial systems, where the Cartesian 4-coordinates select the simultaneity surfaces $x^o = ct = \text{const.}$ as the instantaneous 3-spaces R^3 inside which all the clocks are synchronized with Einstein's convention⁶ and spatial distances are defined as Euclidean distances.

Modern metrology, with also its post-Newtonian extension to general relativity, is reviewed in Refs.[48]: after a conventional definition of a standard of coordinate time, a statement about the value of the round-trip velocity of light c and the choice of Einstein's convention of simultaneity (valid on $x^o = \text{const.}$ hyper-planes in inertial systems and replacing the old slow transport of clocks) is made. Then the derived unit of length (replacing the old *rods*) is defined either as $c \Delta t/2$, where Δt is the round-trip time from the location of the clock to another fixed location (where light is reflected back) or as the wave-length of the radiation emitted from the atomic clock.

After these preliminaries, let us remark that the notion of inertial observer is an idealized

synchronization of the clocks of the observers.

⁵ Its definition implies only one observer and therefore only one clock.

⁶ It is based on the choice of the rays of light as preferred tools to measure time and length. In a given inertial system the clock A , associated to the time-like world-line γ_A , emits a light signal at its time x_{Ai}^o , corresponding to an event Q_i on γ_A , towards the time-like world-line γ_B carrying the clock B . When the signal arrives at a point P on γ_B , it is reflected towards γ_A , where it is detected at time x_{Af}^o , corresponding to an event Q_f on γ_A . Then the clock B at the event P on γ_B is synchronized to the time $x_A^o = \frac{1}{2}(x_{Ai}^o + x_{Af}^o)$, corresponding to an event Q in between Q_i and Q_f . It can be checked that Q and P lie on the same space-like hyper-plane orthogonal to the world-line γ_A , i.e. that they are simultaneous events for the chosen inertial observer.

limit concept: all actual observers are accelerated. Since there is no relativity principle concerning non-inertial observers, their interpretation of experiments relies on the *hypothesis of locality* (see Refs [50, 51, 52]): *an accelerated observer at each instant along its world-line is physically equivalent to an otherwise identical momentarily comoving inertial observer*, namely a non-inertial observer passes through a continuous infinity of hypothetical momentarily comoving inertial observers. While this hypothesis is verified in Newtonian mechanics and in those relativistic cases in which a phenomenon can be reduced to point-like coincidences of classical point particles and light rays (geometrical optic approximation), its validity is questionable in presence of electro-magnetic waves (see Refs.[22, 52]), when the wave-length λ of the radiation under scrutiny, emitted by an accelerated charge, is comparable to the *acceleration length* \mathcal{L} of the observer ⁷.

The fact that we can describe phenomena only locally near the observer and that the actual observers are accelerated leads to the *1+3 point of view* (or threading splitting). Assuming we know the world-line γ of an accelerated observer, we must try to define a notion of simultaneity and 4-coordinates (for instance the Fermi normal ones) around the observer ⁸. However, the knowledge of the observer world-line only allows, in each point of γ , to split the 4-vectors on γ in a part parallel to the observer 4-velocity (the tangent vector to γ) and in a part orthogonal to it. The 3-dimensional orthogonal sub-spaces of the tangent space TM^4 restricted to γ are the *local observer rest frames*: they are taken as a *substitute* of instantaneous simultaneity 3-spaces, orthogonal to the world-line γ , over which to define 3-coordinates. However, this is not a good notion of simultaneous 3-spaces because these hyper-planes intersect each other at a distance of the order of the acceleration length \mathcal{L} of the observer, invalidating the global validity of the Fermi coordinates centered on accelerated observers. Therefore, even if all the locally measured quantities are coordinate-independent tetradic quantities referred to the tetrads associated to the observer, it is not possible to write equations of motion with a well defined Cauchy problem for these tetradic quantities due to the lack of a good notion of simultaneity. As a consequence, statements like the

⁷ $\mathcal{L} = \frac{c^2}{a}$ for an observer with translational acceleration a ; $\mathcal{L} = \frac{c}{\Omega}$ for an observer rotating with frequency Ω . The hypothesis of locality is clearly valid in many Earth-based experiments since $c^2/g_{Earth} \approx 1\text{ }lyr$, $c/\Omega_{Earth} \approx 20\text{ }AU$.

⁸ The observer is assumed endowed with a tetrad, whose time axis is the unit 4-velocity and whose space axes are identified by three orthogonal gyroscopes with a prescribed, but arbitrary, prescription for their transport along the world-line (often the Fermi-Walker transport is preferred due to the associated notion of non-rotation)

conservation of energy cannot be demonstrated using only the 1+3 point of view.

While these problems are less serious in the case of linearly accelerated observers, they become dramatic for rotating ones as it is shown by the enormous number of papers dealing with the *rotating disk* and the *Sagnac effect* (see the bibliography of Refs.[21, 22]). Here we are concerned with congruences of time-like observers [defined by a unit vector field $U^\mu(x)$] which are not synchronizable due to the non-zero vorticity of the congruence: this implies that simultaneity 3-spaces orthogonal to all the world-lines of the observers do not exist and that it is impossible to synchronize the clocks on the rotating disk with Einstein's convention.

This state of affairs implies the necessity of considering the 1+3 point of view as embedded in the dual complementary *3+1 point of view*, in which the starting point is the preliminary introduction of all the possible 3+1 splittings of Minkowski space-time with foliations whose leaves are arbitrary space-like hyper-surfaces and not only space-like hyper-planes. Each of these hyper-surfaces is both a *simultaneity surface*, i.e. a conventional notion of synchronization of distant clocks, and a *Cauchy surface* for the equations of motion of the relativistic system of interest. Each 3+1 splitting has well defined notions of spatial length and of one-way velocity of light.

The 3+1 point of view is less physical (it is impossible to control the initial data on a non-compact space-like Cauchy surface), but it is the only known way to establish a well posed Cauchy problem for the dynamics, so to be able to use the mathematical theorems on the existence and uniqueness of the solutions of field equations for identifying the predictability of the theory. A posteriori, a non-inertial observer can try to separate the part of the dynamics, implied by these solutions, which is determined at each instant from the (assumed known) information coming from its causal past from the part coming from the rest of the universe.

To implement this program we have to come back to Møller's formalization [51] (Chapter VIII, Section 88) of the notion of simultaneity. Given a relativistic inertial system \mathcal{K} with Cartesian 4-coordinates x^μ in Minkowski space-time and with the $x^0 = \text{const.}$ simultaneity hyper-planes, Møller defines the *admissible coordinates transformations* $x^\mu \mapsto y^\mu = f^\mu(x)$ [with inverse transformation $y^\mu \mapsto x^\mu = h^\mu(y)$] as those transformations whose associated metric tensor $g_{\mu\nu}(y) = \frac{\partial h^\alpha(y)}{\partial y^\mu} \frac{\partial h^\beta(y)}{\partial y^\nu} \eta_{\alpha\beta}$ satisfies the following conditions

$$\begin{aligned}
\epsilon g_{oo}(y) &> 0, & \epsilon g_{ii}(y) &< 0, & \left| \begin{array}{cc} g_{ii}(y) & g_{ij}(y) \\ g_{ji}(y) & g_{jj}(y) \end{array} \right| &> 0, \\
\epsilon \det[g_{ij}(y)] &< 0, & \Rightarrow \det[g_{\mu\nu}(y)] &< 0.
\end{aligned} \tag{3.4}$$

These are the necessary and sufficient conditions for having $\frac{\partial h^\mu(y)}{\partial y^o}$ behaving as the velocity field of a relativistic fluid, whose integral curves, the fluid flux lines, are the world-lines of time-like observers. Eqs.(3.4) say:

- i) the observers are time-like because $\epsilon g_{oo} > 0$;
- ii) that the hyper-surfaces $y^o = f^o(x) = \text{const.}$ are good space-like simultaneity surfaces.

Moreover we must ask that $g_{\mu\nu}(y)$ tends to a finite limit at spatial infinity on each of the hyper-surfaces $y^o = f^o(x) = \text{const.}$ If, like in the ADM canonical formulation of metric gravity [53, 54], we write $g_{oo} = \epsilon(N^2 - g_{ij}N^iN^j)$, $g_{oi} = g_{ij}N^j$ introducing the lapse (N) and shift (N^i) functions, this requirement says that the lapse function (i.e. the proper time interval between two nearby simultaneity surfaces) and the shift functions (i.e. the information about which points on two nearby simultaneity surfaces are connected by the so-called *evolution vector field* $\frac{\partial h^\mu(y)}{\partial y^o}$) do not diverge at spatial infinity. This implies that at spatial infinity on each simultaneity surface there is *no asymptotic either translational or rotational acceleration* and this implies that *the simultaneity surfaces must tend to space-like hyper-planes at spatial infinity*.

Let us remark that admissible coordinate transformations $x^\mu \mapsto y^\mu = f^\mu(x)$ constitute the most general extension of the Poincare' transformations $x^\mu \mapsto y^\mu = a^\mu + \Lambda^\mu{}_\nu x^\nu$ compatible with special relativity. A special family of admissible transformations are the *frame-preserving* ones: $x^o \mapsto y^o = f^o(x^o, \vec{x})$, $\vec{x} \mapsto \vec{y} = \vec{f}(\vec{x})$.

It is then convenient to describe [23, 55, 56] the simultaneity surfaces of an admissible foliation (3+1 splitting of Minkowski space-time) with *naturally adapted Lorentz scalar admissible holonomic coordinates* $x^\mu \mapsto \sigma^A = (\tau, \vec{\sigma}) = f^A(x)$ [with inverse $\sigma^A \mapsto x^\mu = z^\mu(\sigma) = z^\mu(\tau, \vec{\sigma})$] such that:

- i) the scalar time coordinate τ labels the leaves Σ_τ of the foliation ($\Sigma_\tau \approx R^3$);
- ii) the scalar curvilinear 3-coordinates $\vec{\sigma} = \{\sigma^r\}$ on each Σ_τ are defined with respect to an arbitrary time-like centroid $x^\mu(\tau)$ chosen as their origin;
- iii) if $y^\mu = f^\mu(x)$ is any admissible coordinate transformation describing the same foliation, i.e. if the leaves Σ_τ are also described by $y^o = f^o(x) = \text{const.}$, then, modulo

reparametrizations, we must have $y^\mu = f^\mu(z(\tau, \vec{\sigma})) = \tilde{f}^\mu(\tau, \vec{\sigma}) = A^\mu_A \sigma^A$ with $A^o_\tau = \text{const.}$, $A^o_r = 0$, so that we get $y^o = \text{const.}$, $y^i = A^i_A(\tau, \vec{\sigma}) \sigma^A$. The τ and $\vec{\sigma}$ adapted admissible coordinates may be called *radar-like 4-coordinates* with respect to the arbitrary non-inertial observer, whose world-line $x^\mu(\tau) = z^\mu(\tau, \vec{0})$ is chosen as origin of the 3-coordinates: since the 3-surfaces Σ_τ are *not orthogonal* to this world-line, the pathologies of the Fermi coordinates are avoided. Therefore these foliations describe possible notions of simultaneity for the non-inertial observer $x^\mu(\tau)$.

The use of these Lorentz-scalar adapted coordinates allows to make statements depending only on the foliation but not on the 4-coordinates y^μ used for Minkowski space-time.

The simultaneity hyper-surfaces Σ_τ are described by their embedding $x^\mu = z^\mu(\tau, \vec{\sigma})$ in Minkowski space-time $[R^3 \mapsto \Sigma_\tau \subset M^4 \approx R \times R^3, (\tau, \vec{\sigma}) \mapsto z^\mu(\tau, \vec{\sigma})]$ and the induced metric is $g_{AB}(\tau, \vec{\sigma}) = z^\mu_A(\tau, \vec{\sigma}) z^\nu_B(\tau, \vec{\sigma}) \eta_{\mu\nu}$ with $z^\mu_A = \partial z^\mu / \partial \sigma^A$ (they are flat tetrad fields over Minkowski space-time). Since the vector fields $z^\mu_r(\tau, \vec{\sigma})$ are tangent to the surfaces Σ_τ , the time-like vector field of normals $l^\mu(\tau, \vec{\sigma})$ is proportional to $\epsilon^\mu_{\alpha\beta\gamma} z^\alpha_1(\tau, \vec{\sigma}) z^\beta_2(\tau, \vec{\sigma}) z^\gamma_3(\tau, \vec{\sigma})$. Instead the time-like evolution vector field is $z^\mu_\tau(\tau, \vec{\sigma}) = N(\tau, \vec{\sigma}) l^\mu(\tau, \vec{\sigma}) + N^r(\tau, \vec{\sigma}) z^\mu_r(\tau, \vec{\sigma})$, so that we have $dz^\mu(\tau, \vec{\sigma}) = z^\mu_\tau(\tau, \vec{\sigma}) d\tau + z^\mu_r(\tau, \vec{\sigma}) d\sigma^r = N(\tau, \vec{\sigma}) d\tau l^\mu(\tau, \vec{\sigma}) + (N^r(\tau, \vec{\sigma}) d\tau + d\sigma^r) z^\mu_r(\tau, \vec{\sigma})$.

Since the 3-surfaces Σ_τ are *equal time* 3-spaces with all the clocks synchronized, the spatial distance between two equal-time events will be $dl_{12} = \int_1^2 dl \sqrt{{}^3g_{rs}(\tau, \vec{\sigma}(l)) \frac{d\sigma^r(l)}{dl} \frac{d\sigma^s(l)}{dl}} [\vec{\sigma}(l)]$ is a parametrization of the 3-geodesic γ_{12} joining the two events on Σ_τ . Moreover, by using test rays of light we can define the *one-way* velocity of light between events on different Σ_τ 's.

The main property of *each foliation with simultaneity surfaces* associated to an admissible 4-coordinate transformation is that the embedding of the leaves of the foliation automatically determine two time-like vector fields and therefore *two congruences of (in general) non-inertial time-like observers*:

i) The time-like vector field $l^\mu(\tau, \vec{\sigma})$ of the normals to the simultaneity surfaces Σ_τ (by construction surface-forming, i.e. irrotational), whose flux lines are the world-lines of the so-called (in general non-inertial) Eulerian observers. The simultaneity surfaces Σ_τ are (in general non-flat) Riemannian 3-spaces in which the physical system is visualized and in each point the tangent space to Σ_τ is the local observer rest frame $R_{l(\tau_\gamma)}$ of the Eulerian observer through that point. This 3+1 viewpoint is called *hyper-surface 3+1 splitting*.

ii) The time-like evolution vector field $z^\mu_\tau(\tau, \vec{\sigma}) / \sqrt{\epsilon g_{\tau\tau}(\tau, \vec{\sigma})}$, which in general is not

surface-forming (i.e. it has non-zero vorticity like in the case of the rotating disk). The observers associated to its flux lines have the local observer rest frames $R_{\vec{u}(\tau_\gamma)}$ not tangent to Σ_τ : there is no intrinsic notion of instantaneous 3-space for these observers (1+3 point of view or *threading splitting*) and no visualization of the physical system in large. However these observers can use the notion of simultaneity associated to the embedding $z^\mu(\tau, \vec{\sigma})$, which determines their 4-velocity. This 3+1 viewpoint is called *slicing 3+1 splitting*. In the case of the uniformly rotating disk all the existing rotating 4-coordinate systems have a *coordinate singularity* ($g_{oo}(y^o, \vec{y}) = 0$) where $\omega r = c$: there the time-like observers of the congruence would become null observers like on the horizon of a Schartzschild black hole and this is not acceptable in absence of a horizon.

As shown in Ref.[22] the 3+1 point of view allows to get the following results:

i) To define the special class of foliations implementing the idea behind the locality hypothesis, that a non-inertial observer is equivalent to a continuous family of comoving inertial observers, but with restrictions coming from the admissibility conditions. The main byproduct of these restrictions will be that there exist admissible 4-coordinate transformations interpretable as *rigid systems of reference with arbitrary translational acceleration*. However there is *no admissible 4-coordinate transformation corresponding to a rigid system of reference with rotational motion*. When rotations are present, the admissible 4-coordinate transformations give rise to a continuum of local systems of reference like it happens in general relativity (*differential rotations*).

ii) The simplest foliation of the previous class, whose simultaneity surfaces are space-like hyper-planes with differentially rotating 3-coordinates is given by the embedding

$$z^\mu(\tau, \vec{\sigma}) = x^\mu(\tau) + \epsilon_r^\mu R_s^r(\tau, \sigma) \sigma^s \stackrel{def}{=} x^\mu(\tau) + b_r^\mu(\tau, \sigma) \sigma^r,$$

$$R_s^r(\tau, \sigma) \rightarrow_{\sigma \rightarrow \infty} \delta_s^r, \quad \partial_A R_s^r(\tau, \sigma) \rightarrow_{\sigma \rightarrow \infty} 0,$$

$$b_s^\mu(\tau, \sigma) = \epsilon_r^\mu R_s^r(\tau, \sigma) \rightarrow_{\sigma \rightarrow \infty} \epsilon_s^\mu, \quad [b_r^\mu \eta_{\mu\nu} b_s^\nu](\tau, \sigma) = -\epsilon \delta_{rs},$$

$$R = R(\alpha, \beta, \gamma), \quad \text{with Euler angles satisfying}$$

$$\alpha(\tau, \sigma) = F(\sigma) \tilde{\alpha}(\tau), \quad \beta(\tau, \sigma) = F(\sigma) \tilde{\beta}(\tau), \quad \gamma(\tau, \sigma) = F(\sigma) \tilde{\gamma}(\tau),$$

$$0 < F(\sigma) < \frac{m}{2K M_1 \sigma} (K - 1) = \frac{1}{M \sigma}, \quad \frac{dF(\sigma)}{d\sigma} \neq 0,$$

$$\text{or} \quad |\partial_\tau \alpha(\tau, \sigma)|, |\partial_\tau \beta(\tau, \sigma)|, |\partial_\tau \gamma(\tau, \sigma)| < \frac{m}{2K \sigma} (K - 1). \quad (3.5)$$

iii) To solve the following inverse problem: given a time-like unit vector field, i.e. a (in general not irrotational) congruence of non-inertial observers like that associated with a rotating disk, find an admissible foliation with simultaneity surfaces such that $z^\mu_\tau(\tau, \vec{\sigma})$ is proportional to the given vector field.

iv) To define an operational method, generalizing Einstein's convention from inertial hyper-planes to arbitrary admissible simultaneity surfaces. It can be used to build a grid of radar 4-coordinates to be used by a set of satellites of the Global-Positioning-System type.

v) To give the 3+1 point of view on the rotating disk and the Sagnac effect by using the embedding (3.5) as an example. Now there are a foliation-dependent instantaneous 3-space and a foliation-dependent 3-geometry for the rotating disk.

vi) To use the foliation (3.5) to describe Earth rotation in the determination of the one-way time transfer for the propagation of light from an Earth station to a satellite, with the consequence that the ESA ACES mission on the synchronization of clocks [58] can be re-interpreted as a measure of the deviation of this admissible simultaneity convention from Einstein's one.

vii) To use the description of electro-magnetism as a parametrized Minkowski theory to arrive at Maxwell equations in non-inertial frames.

In conclusion, the absence of an absolute simultaneity and of an absolute notion of instantaneous 3-space, replaced by the absolute chrono-geometrical structure of Minkowski space-time, forces every time-like observer to choose an admissible 3+1 splitting of it to formulate a theory of measurement and a well posed Cauchy problem for the dynamics. As we have seen this can be done in many ways, which generalize Einstein's convention for inertial observers (the traditional foliation with the hyper-planes $x^o = \text{const.}$) by relaxing the condition that the observer's world-line is orthogonal to the *equal time* 3-spaces Σ_τ . Then the new problem is whether all these possible notions of simultaneity lead to an equivalent description of phenomena. In the next Subsection we introduce parametrized Minkowski theories, in which this equivalence is realized as a *gauge equivalence* in the sense of Dirac theory of constraints.

C. Parametrized Minkowski Theories.

The previous two Subsections justify the attempt [55] (see also Refs.[56] and Appendix A of Ref.[54]) to reformulate every isolated system on arbitrary space-like *equal time* 3-surfaces Σ_τ leaves of an admissible 3+1 splitting of Minkowski space-time, so to get a parametrized field theory already in a form suited to the transition to general relativity in its ADM canonical formulation. The starting point was given by Dirac [11] with his reformulation of classical field theory on space-like hyper-surfaces foliating Minkowski space-time M^4 .

If $z^\mu(\tau, \vec{\sigma})$ is the embedding of the leaves Σ_τ , particle world-lines $x_i^\mu(\tau) = z^\mu(\tau, \vec{\eta}_i(\tau))$ are identified by scalar 3-coordinates $\vec{\eta}_i(\tau)$ labeling the intersection of the world-lines with Σ_τ with respect to the centroid $x^\mu(\tau) = z^\mu(\tau, \vec{0})$ chosen as origin. This solves the problem of the relative times (all the particles are at the *same* scalar time τ), forcing us to solve the mass-shell constraints $p_i^2 - m_i^2 \approx 0$ (in the free case) and to choose the sign of the energy $p_i^o = \pm \sqrt{m_i^2 + \vec{p}_i^2}$ of each particle. In this way we get a $3N$ -dimensional configuration space for the N -particle system without mass-shell constraints. For a scalar field we replace the traditional $\tilde{\phi}(x^o, \vec{x})$ with the new field $\phi(\tau, \vec{\sigma}) = \tilde{\phi}(z(\tau, \vec{\sigma}))$ which *knows* the non-local information of the chosen simultaneity, being a function of the adapted radar 4-coordinates. For the electro-magnetic field we use $A_A(\tau, \vec{\sigma}) = \frac{\partial z^\mu(\tau, \vec{\sigma})}{\partial \sigma^A} A_\mu(z(\tau, \vec{\sigma}))$ and so on. All the new fields are Lorentz scalar, having only surface indices, satisfy (if any) Lorentz scalar constraints and, due to the absence of the mass-shell constraints, the problem quoted at the end of the Subsection on relativistic mechanics disappears. This treatment of the fields is the classical basis of the Tomonaga-Schwinger quantum field theory [47].

Then one rewrites the Lagrangian density of the given isolated system in the form required by the minimal coupling to an external gravitational field. Instead of considering the 4-metric as describing a gravitational field, here one replaces the 4-metric with the induced metric $g_{AB}[z] = z_A^\mu \eta_{\mu\nu} z_B^\nu$ on Σ_τ , which is a functional of z^μ and considers the embedding $z^\mu(\tau, \vec{\sigma})$ as *new configurational fields*. In this way the Lagrangian density becomes a functional of the embedding and of the isolated system.

The action of the system $S = \int d\tau d^3\sigma \mathcal{L}(\tau, \vec{\sigma})$ [$\mathcal{L}(\tau, \vec{\sigma}) = \pm m \delta^3(\vec{\sigma} - \vec{\eta}(\tau)) \sqrt{g_{\tau\tau}(\tau, \vec{\sigma}) + 2g_{\tau r}(\tau, \vec{\sigma}) \eta^r(\tau) + r_{rs}(\tau, \vec{\sigma}) \eta^r(\tau) \eta^s(\tau)}$ for a scalar particle of mass $\pm m$] is invariant under frame-preserving reparametrizations: $\tau \mapsto \tau'(\tau, \vec{\sigma})$ and $\vec{\sigma} \mapsto \vec{\sigma}'(\vec{\sigma})$. This is a *non-trivial special relativistic type of general covariance* implying that the embedding configuration variables $z^\mu(\tau, \vec{\sigma})$ are *gauge variables*, so that *the physical results about the system do not depend on the choice of the notion of simultaneity*. Therefore, in parametrized Minkowski theories the *conventionalism of simultaneity* is rephrased as a *gauge problem*, i.e. as the *arbitrary choice of a gauge fixing selecting a well defined notion of simultaneity among those allowed by the gauge freedom*.

From this Lagrangian, besides a Lorentz-scalar form of the constraints of the given system, we get four extra primary first class constraints

$$\begin{aligned} \mathcal{H}_\mu(\tau, \vec{\sigma}) &= \rho_\mu(\tau, \vec{\sigma}) - l_\mu(\tau, \vec{\sigma}) T_{sys}^{\tau\tau}(\tau, \vec{\sigma}) - z_{r\mu}(\tau, \vec{\sigma}) T_{sys}^{\tau r}(\tau, \vec{\sigma}) \approx 0, \\ \{\mathcal{H}_\mu(\tau, \vec{\sigma}), \mathcal{H}_\nu(\tau, \vec{\sigma}')\} &= 0. \end{aligned} \quad (3.6)$$

Here $T_{sys}^{\tau\tau}(\tau, \vec{\sigma})$, $T_{sys}^{\tau r}(\tau, \vec{\sigma})$, are the components of the energy-momentum tensor in the holonomic coordinate system, corresponding to the energy- and momentum-density of the isolated system, $\rho_\mu(\tau, \vec{\sigma})$ is the canonical momentum conjugate to $z^\mu(\tau, \vec{\sigma})$ and $l^\mu(\tau, \vec{\sigma})$ is the unit normal to Σ_τ . These constraints are the generators of Hamiltonian gauge transformations implying the independence of the description from the choice of the 3+1 splitting, i.e. from the choice of the foliation with space-like hyper-surfaces. The evolution vector is given by $z_\tau^\mu = N_{[z](flat)} l^\mu + N_{[z](flat)}^r z_r^\mu$ and $N_{[z](flat)}(\tau, \vec{\sigma})$, $N_{[z](flat)}^r(\tau, \vec{\sigma})$ are the flat lapse and shift functions defined through the metric like in general relativity: however, now they are not independent variables but functionals of $z^\mu(\tau, \vec{\sigma})$.

The Dirac Hamiltonian contains the piece $\int d^3\sigma \lambda^\mu(\tau, \vec{\sigma}) \mathcal{H}_\mu(\tau, \vec{\sigma})$ with $\lambda^\mu(\tau, \vec{\sigma})$ Dirac multipliers. It is possible to rewrite the integrand in the form [${}^3g^{rs}$ is the inverse of g_{rs}]

$$\begin{aligned}\lambda_\mu(\tau, \vec{\sigma}) \mathcal{H}^\mu(\tau, \vec{\sigma}) &= [(\lambda_\mu l^\mu)(l_\nu \mathcal{H}^\nu) - (\lambda_\mu z_r^\mu)({}^3g^{rs} z_{s\nu} \mathcal{H}^\nu)](\tau, \vec{\sigma}) = \\ &\stackrel{def}{=} N_{(flat)}(\tau, \vec{\sigma})(l_\mu \mathcal{H}^\mu)(\tau, \vec{\sigma}) - N_{(flat)r}(\tau, \vec{\sigma})({}^3g^{rs} z_{s\mu} \mathcal{H}^\mu)(\tau, \vec{\sigma}),\end{aligned}\quad (3.7)$$

with the (nonholonomic form of the) constraints $(l_\mu \mathcal{H}^\mu)(\tau, \vec{\sigma}) \approx 0$, $({}^3g^{rs} z_{s\mu} \mathcal{H}^\mu)(\tau, \vec{\sigma}) \approx 0$, satisfying the universal Dirac algebra of the ADM constraints of canonical metric gravity. In this way we have defined new flat lapse and shift functions $N_{(flat)}(\tau, \vec{\sigma}) = \lambda_\mu(\tau, \vec{\sigma}) l^\mu(\tau, \vec{\sigma})$, $N_{(flat)r}(\tau, \vec{\sigma}) = \lambda_\mu(\tau, \vec{\sigma}) z_r^\mu(\tau, \vec{\sigma})$, which have the same content of the arbitrary Dirac multipliers $\lambda_\mu(\tau, \vec{\sigma})$, namely they multiply primary first class constraints satisfying the Dirac algebra. In Minkowski space-time they are quite distinct from the previous lapse and shift functions $N_{[z](flat)}$, $N_{[z](flat)r}$, defined starting from the metric.

In special relativity, it is convenient to restrict ourselves to arbitrary space-like hyperplanes $z^\mu(\tau, \vec{\sigma}) = x_s^\mu(\tau) + b_r^\mu(\tau) \sigma^r$. Since they are described by only 10 variables, after this restriction we remain only with 10 first class constraints determining the 10 variables conjugate to the hyperplane in terms of the variables of the system:

$$\mathcal{H}^\mu(\tau) = p_s^\mu - p_{(sys)}^\mu \approx 0, \quad \mathcal{H}^{\mu\nu}(\tau) = S_s^{\mu\nu} - S_{(sys)}^{\mu\nu} \approx 0. \quad (3.8)$$

After the restriction to space-like hyper-planes the previous piece of the Dirac Hamiltonian is reduced to $\tilde{\lambda}^\mu(\tau) \mathcal{H}_\mu(\tau) - \frac{1}{2} \tilde{\lambda}^{\mu\nu}(\tau) \mathcal{H}_{\mu\nu}(\tau)$. Since at this stage we have $z_r^\mu(\tau, \vec{\sigma}) \approx b_r^\mu(\tau)$, so that $z_r^\mu(\tau, \vec{\sigma}) \approx N_{[z](flat)}(\tau, \vec{\sigma}) l^\mu(\tau, \vec{\sigma}) + N_{[z](flat)r}^r(\tau, \vec{\sigma}) b_r^\mu(\tau, \vec{\sigma}) \approx \dot{x}_s^\mu(\tau) + \dot{b}_r^\mu(\tau) \sigma^r = -\tilde{\lambda}^\mu(\tau) - \tilde{\lambda}^{\mu\nu}(\tau) b_{r\nu}(\tau) \sigma^r$, it is only now that we get the coincidence of the two definitions of flat lapse and shift functions:

$$\begin{aligned}N_{[z](flat)}(\tau, \vec{\sigma}) &\approx N_{(flat)}(\tau, \vec{\sigma}) = -\tilde{\lambda}_\mu(\tau) l^\mu - l^\mu \tilde{\lambda}_{\mu\nu}(\tau) b_s^\nu(\tau) \sigma^s, \\ N_{[z](flat)r}(\tau, \vec{\sigma}) &\approx N_{(flat)r}(\tau, \vec{\sigma}) = -\tilde{\lambda}_\mu(\tau) b_r^\mu(\tau) - b_r^\mu(\tau) \tilde{\lambda}_{\mu\nu}(\tau) b_s^\nu(\tau) \sigma^s.\end{aligned}\quad (3.9)$$

The 20 variables for the phase space description of a hyperplane are:

i) $x_s^\mu(\tau), p_s^\mu$, [or $T_s = p_s \cdot \tilde{x}_s / \epsilon_s$, $\epsilon_s = \sqrt{\epsilon p_s^2}$, \vec{z}_s , \vec{k}_s] parametrizing the origin of the coordinates on the family of space-like hyper-planes. The four constraints $\mathcal{H}^\mu(\tau) \approx 0$ say that p_s^μ is determined by the 4-momentum of the isolated system.

ii) $b_A^\mu(\tau)$ (with the $b_r^\mu(\tau)$'s being three orthogonal space-like unit vectors generating the time-like unit normal $b_r^\mu(\tau) = l^\mu(\tau)$ to the hyper-planes) and a spin tensor $S_s^{\mu\nu} = -S_s^{\nu\mu}$ with the orthonormality constraints $b_A^\mu {}^4\eta_{\mu\nu} b_B^\nu = {}^4\eta_{AB}$ ⁹. In these variables there are hidden six independent pairs of degrees of freedom. The six constraints $\mathcal{H}^{\mu\nu}(\tau) \approx 0$ say that $S_s^{\mu\nu}$ coincides the spin tensor of the isolated system. Then one has that p_s^μ , $J_s^{\mu\nu} = x_s^\mu p_s^\nu - x_s^\nu p_s^\mu + S_s^{\mu\nu}$, satisfy the algebra of the Poincaré group. Finally the requirement of a τ -independent normal l^μ is equivalent to three more gauge fixings, forbidding the action of Lorentz boosts, and reducing to seven the surviving first class constraints.

Let us remark that after the restriction to these hyper-planes with constant normal the only surviving congruence of time-like observers (see Subsection B) is composed by inertial observers having the unit normal as 4-velocity. Instead, with more general admissible 3+1 splittings, we have two congruences of non-inertial observers and in Ref.[22] there is a preliminary study of their description of the dynamics using the electro-magnetic field as an example.

D. The Rest-Frame Wigner-Covariant Instant Form.

Let us remark that, for each configuration of an isolated system there is a privileged family of hyper-planes (the *Wigner hyper-planes orthogonal to p_s^μ* , existing when $\epsilon p_s^2 > 0$), namely a *preferred notion of simultaneity*, corresponding to the *intrinsic rest-frame of the isolated system* [55]. If we choose these hyper-planes with suitable gauge fixings, we remain with only the four constraints $\mathcal{H}^\mu(\tau) \approx 0$, which can be rewritten as

$$\begin{aligned}\epsilon_s &\approx [\textit{invariant mass of the isolated system under investigation}] = M_{sys}, \\ \vec{p}_{sys} &= [3 - \textit{momentum of the isolated system inside the Wigner hyperplane}] \approx 0.\end{aligned}\tag{3.10}$$

There is no more a restriction on p_s^μ , because $u_s^\mu(p_s) = p_s^\mu/\epsilon_s$ gives the orientation of the Wigner hyper-planes containing the isolated system with respect to an arbitrary given external inertial observer.

⁹ Enforced by assuming the Dirac brackets $\{S_s^{\mu\nu}, b_A^\rho\} = {}^4\eta^{\rho\nu} b_A^\mu - {}^4\eta^{\rho\mu} b_A^\nu$, $\{S_s^{\mu\nu}, S_s^{\alpha\beta}\} = C_{\gamma\delta}^{\mu\nu\alpha\beta} S_s^{\gamma\delta}$ with $C_{\gamma\delta}^{\mu\nu\alpha\beta}$ the structure constants of the Lorentz algebra.

In this special gauge we have $b_A^\mu \equiv L^\mu_A(p_s, \overset{\circ}{p}_s)$ (the standard Wigner boost for time-like Poincaré orbits), $S_s^{\mu\nu} \equiv S_{system}^{\mu\nu}$, and the only remaining canonical variables are the non-covariant canonical coordinate $\tilde{x}_s^\mu(\tau)$ (living on the Wigner hyper-planes) and p_s^μ . The embedding for the Wigner hyper-planes is $z^\mu(\tau, \vec{\sigma}) = x^\mu(0) + b_A^\mu \sigma^A$. Now 3 degrees of freedom of the isolated system (an *internal* center-of-mass 3-variable $\vec{\sigma}_{sys}$ defined inside the Wigner hyperplane and conjugate to \vec{p}_{sys}) become gauge variables¹⁰, while the \tilde{x}^μ is playing the role of a kinematical Newton-Wigner-like *external* 4-center of mass for the isolated system and may be interpreted as a decoupled observer with his parametrized clock (*point particle clock*). All the fields living on the Wigner hyperplane are now either Lorentz scalar or with their 3-indices transforming under Wigner rotations (induced by Lorentz transformations in Minkowski space-time) as any Wigner spin 1 index.

One obtains in this way a new kind of instant form of the dynamics (see Ref.[38]), the *Wigner-covariant 1-time rest-frame instant form* [55] with a universal breaking of Lorentz covariance. It is the special relativistic generalization of the non-relativistic separation of the center of mass from the relative motion [$H = \frac{\vec{P}^2}{2M} + H_{rel}$]. The role of the center of mass is taken by the Wigner hyperplane, identified by the point $\tilde{x}^\mu(\tau)$ and by its normal p_s^μ . The invariant mass M_{sys} of the system replaces the non-relativistic Hamiltonian H_{rel} for the relative degrees of freedom, after the addition of the gauge-fixing $T_s - \tau \approx 0$ ($T_s = p_s \cdot \tilde{x}_s / \epsilon_s = p_s \cdot x_s / \epsilon_s$) and generates the evolution in this rest-frame time.

The Wigner hyperplane with its natural Euclidean metric structure offers a natural solution to the problem of boost for lattice gauge theories and realizes explicitly the machian aspect of dynamics that only relative motions are relevant.

The isolated systems till now analyzed to get their rest-frame Wigner-covariant generalized Coulomb gauges, i.e. the subset of global Shanmugadhasan canonical bases, which, for each Poincaré stratum, are also adapted to the geometry of the corresponding Poincaré orbits with their little groups, are:

a) The system of N scalar particles with Grassmann electric charges plus the electromagnetic field [55]. The final Dirac's observables are: i) the transverse radiation field variables $\vec{A}_\perp, \vec{E}_\perp$; ii) the particle canonical variables $\vec{\eta}_i(\tau), \vec{\kappa}_i(\tau)$, dressed with a Coulomb cloud. The physical Hamiltonian contains the mutual instantaneous Coulomb potentials extracted from

¹⁰ The natural gauge fixing is $\vec{\sigma}_{sys} \approx 0$, so that it coincides with the centroid $x_s^\mu(\tau) = z^\mu(\tau, \vec{\sigma} = 0)$ origin of the 3-coordinates on the Wigner hyper-plane.

field theory and there is a regularization of the Coulomb self-energies due to the Grassmann character of the electric charges Q_i [$Q_i^2 = 0$, $Q_i Q_j = Q_j Q_i \neq 0$ for $i \neq j$]. In Ref.[57] there is the study of the Lienard-Wiechert potentials and of Abraham-Lorentz-Dirac equations in this rest-frame Coulomb gauge. In the semi-classical approximation of Ref.[56], the electromagnetic degrees of freedom are re-expressed in terms of the particle variables by means of the Lienard-Wiechert solution in the framework of the rest-frame instant form. In this way it has been possible to derive the exact semi-classical relativistic form of the action-at-a-distance Darwin potential (or the Salpeter one for spinning particles) in the reduced phase space of the particles. Note that these potentials, till now deduced only from quantum field theory through the Bethe-Salpeter equation, are independent of the choice of the Green function in the Lienard-Wiechert solution due to the semi-classical regularization.

Also the rest-frame 1-time relativistic statistical mechanics has been developed [55].

b) The system of N scalar particles with Grassmann-valued color charges plus the color $SU(3)$ Yang-Mills field[59]: it gives the pseudoclassical description of the relativistic scalar-quark model, deduced from the classical QCD Lagrangian and with the color field present. The physical invariant mass of the system is given in terms of the Dirac observables. From the reduced Hamilton equations the second order equations of motion both for the reduced transverse color field and the particles are extracted. Then, one studies the $N=2$ (meson) case. A special form of the requirement of having only color singlets, suited for a field-independent quark model, produces a *pseudoclassical asymptotic freedom* and a regularization of the quark self-energy. With these results one can covariantize the bosonic part of the standard model given in Ref.[34].

c) The system of N spinning particles of definite energy $[(\frac{1}{2}, 0)$ or $(0, \frac{1}{2})$ representation of $SL(2, C)$] with Grassmann electric charges plus the electromagnetic field[60] and that of a Grassmann-valued Dirac field plus the electromagnetic field (the pseudoclassical basis of QED) [61].

d) Relativistic perfect fluids have been reformulated [62] as parametrized Minkowski theories and their rest-frame instant form is known.

In conclusion all the fields (Klein-Gordon, Yang-Mills and Dirac) appearing in the $SU(3) \times SU(2) \times U(1)$ model of elementary particles have been reformulated as parametrized Minkowski theories.

E. Relativistic Kinematics and the Møller Radius.

The formulations of relativistic mechanics with first class constraints in a $8N$ -dimensional phase space $x_i^\mu(\tau)$, $p_i^\mu(\tau)$ (see Subsection A) led to discover a quasi-Shanmugadhasan canonical transformation for the N -body problem adapted to the $N - 1$ constraints $\bar{\phi}_a \approx 0$ and having $N - 1$ Lorentz-scalar relative times (generalizing $T_R = p \cdot r / \epsilon$ of the $N = 2$ case) as conjugate gauge variables. This adapted basis [55, 64] also contains: i) a pair $T = p \cdot \tilde{x} / \epsilon$, $\epsilon = \sqrt{\epsilon p^2}$ (with the mass-spectrum ϵ to be determined from the constraint $\bar{\phi}_+ \approx 0$); ii) a decoupled non-covariant 3-center of mass \vec{z} and its conjugate momentum \vec{k} ; iii) $N - 1$ pairs of relative variables $\vec{\rho}_a$, $\vec{\pi}_a$ (Dirac observables gauge invariant with respect to the gauge transformations generated by the $\bar{\phi}_a$'s). Since $H_c \equiv 0$, on each branch of the mass spectrum $\vec{\rho}_a$, $\vec{\pi}_a$ can be replaced with an equal number of constant Jacobi data representing the true Dirac observables of the frozen picture associated with the reduced phase space of these reparametrization invariant theories, by means of a branch-dependent Shanmugadhasan canonical transformation ¹¹.

The understanding of this type of N -body kinematics was of help in developing a new relativistic kinematics [64] adapted to the framework of parametrized Minkowski theories and in particular to the rest-frame instant form of dynamics. For a positive-energy N -body system we start with a $6N$ -dimensional phase space $\vec{\eta}_i(\tau)$, $\vec{\kappa}_i(\tau)$ [$x_i^\mu(\tau) = z^\mu(\tau, \vec{\eta}_i(\tau))$ and $p_i^\mu(\tau)$ ($p_i^2 = m_i^2$, $sign p_i^0 = +$) are derived dependent quantities] and we can study the canonical transformations to a new canonical basis containing a canonical *internal* 3-center of mass $\vec{\sigma}_{sys}$ conjugate to $\vec{p}_{sys} \approx 0$. The rest-frame instant form leads to a *doubling of viewpoints and concepts*:

1) The *external* viewpoint, taken by an arbitrary inertial Lorentz observer, who describes the Wigner hyper-planes determined by the time-like configurations of the isolated system. A change of inertial observer by means of a Lorentz transformation rotates the Wigner hyper-planes and induces a Wigner rotation of the 3-vectors inside each Wigner hyperplane. Every such hyperplane inherits an induced *internal Euclidean structure* while an *external* realization of the Poincaré group induces an *internal* Euclidean action.

2) The *internal* viewpoint, taken by an observer inside the Wigner hyper-planes. This viewpoint is associated to a unfaithful *internal* realization of the Poincaré algebra: the total

¹¹ In the free case it can be done. In general it can be done every time the dynamics is Liouville integrable.

internal 3-momentum of the isolated system vanishes due to the rest-frame conditions. The *internal* energy and angular momentum are the invariant mass M_{sys} and the spin S_{sys}^{rs} (the angular momentum with respect to $\tilde{x}_s^\mu(\tau)$) of the isolated system, respectively.

The determination of $\vec{\sigma}_{sys}$ may be done with the group theoretical methods of Ref.[65]: given a realization on the phase space of a given system of the ten Poincaré generators one can build three 3-position variables only in terms of them. In our case of a system on the Wigner hyperplane with $\vec{p}_{sys} \approx 0$ and using the internal Poincaré' algebra they are: i) a canonical center of mass (the *internal* center of mass $\vec{\sigma}_{sys}$); ii) a non-canonical Møller center of energy $\vec{\sigma}_{sys}^{(E)}$; iii) a non-canonical Fokker-Pryce center of inertia $\vec{\sigma}_{sys}^{(FP)}$. Due to $\vec{p}_{sys} \approx 0$, we have $\vec{\sigma}_{sys} \approx \vec{\sigma}_{sys}^{(E)} \approx \vec{\sigma}_{sys}^{(FP)}$. By adding the gauge fixings $\vec{\sigma}_{sys} \approx 0$ one can show that the origin $x_s^\mu(\tau)$ becomes simultaneously the Dixon center of mass of an extended object and both the Pirani and Tulczyjew centroids. With similar methods, starting from the external Poincaré' algebra, one can construct three *external* collective positions (all located on the Wigner hyper-plane): i) the *external* canonical non-covariant center of mass \tilde{x}_s^μ ; ii) the *external* non-canonical and non-covariant Møller center of energy R_s^μ ; iii) the *external* covariant non-canonical Fokker-Pryce center of inertia Y_s^μ (when there are the gauge fixings $\vec{\sigma}_{sys} \approx 0$ it also coincides with the centroid x_s^μ origin of the 3-coordinates).

In the gauge where $\epsilon_s \equiv M_{sys}$, $T_s \equiv \tau$, the canonical basis $\vec{z}_s, \vec{k}_s, \vec{\eta}_i, \vec{\kappa}_i$ is restricted by the three pairs of second class constraints $\vec{\kappa}_+ = \sum_{i=1}^N \vec{\kappa}_i \approx 0$ (the rest-frame condition), $\vec{\sigma}_{sys} \approx 0$, so that 6N canonical variables describe the N particles like in the non-relativistic case. We still need a canonical transformation $\vec{\eta}_i, \vec{\kappa}_i \mapsto \vec{\sigma}_{sys}[\approx 0], \vec{\kappa}_+[\approx 0], \vec{\rho}_a, \vec{\pi}_a$ [$a = 1, \dots, N-1$] identifying a set of *relative canonical variables*. The final 6N-dimensional canonical basis is $\vec{z}_s, \vec{k}_s, \vec{\rho}_a, \vec{\pi}_a$. To get this result we need a highly non-linear (but point in the momenta) canonical transformation[64], which can be obtained by exploiting the Gartenhaus-Schwartz singular transformation [66].

At the end we obtain *the Hamiltonian for the relative motions as a sum of N square roots*, each one containing a squared mass and a quadratic form in the relative momenta, which goes into the non-relativistic Hamiltonian for relative motions in the limit $c \rightarrow \infty$.

The N quadratic forms in the relative momenta appearing in the relative Hamiltonian *cannot be simultaneously diagonalized* and it can be shown that concepts like *reduced masses*, *Jacobi normal relative coordinates* and *tensor of inertia* cannot be extended to special relativity. Instead in the non-relativistic N-body problem the fact that the non-relativistic

kinetic energy of the relative motions is a quadratic form in the relative velocities allows the introduction of special sets of relative coordinates, the *Jacobi normal relative coordinates* that diagonalize the quadratic form and correspond to different patterns of clustering of the centers of mass of the particles. Each set of Jacobi normal relative coordinates organizes the N particles into a *hierarchy of clusters*, in which each cluster of two or more particles has a mass given by an eigenvalue (*reduced masses*) of the quadratic form; Jacobi normal coordinates join the centers of mass of pairs of clusters.

Moreover, the non-Abelian nature of the rotation symmetry group whose associated Noether constants of motion (the conserved total angular momentum) are not in involution, prevents the possibility of a global separation of absolute rotations from the relative motions, so that there is no global definition of absolute *vibrations*. Consequently, an *isolated* deformable body can undergo rotations by changing its own shape (see the examples of the *falling cat* and of the *diver*). It was just to deal with these problems that the theory of the orientation-shape $SO(3)$ principal bundle approach[67] has been developed. Its essential content is that any *static* (i.e. velocity-independent) definition of *body frame* for a deformable body must be interpreted as a gauge fixing in the context of a $SO(3)$ *gauge* theory. Both the laboratory and the body frame angular velocities, as well as the orientational variables of the static body frame, become thereby *unobservable gauge* variables. This approach is associated with a set of *point* canonical transformations, which allow to define the body frame components of relative motions in a velocity-independent way.

Since in many physical applications (e.g. nuclear physics, rotating stars,...) angular velocities are viewed as *measurable* quantities, one would like to have an alternative formulation complying with this requirement and possibly generalizable to special relativity. This has been done in Ref.[68] starting from the canonical basis $\vec{\rho}_a, \vec{\pi}_a$. First of all, for $N \geq 3$, we have constructed a class of *non-point* canonical transformations which allow to build the so called *canonical spin bases*: they are connected to the patterns of the possible *clusterings of the spins* associated with relative motions. The definition of these *spin bases* is independent of Jacobi normal relative coordinates, just as the patterns of spin clustering are independent of the patterns of center-of-mass Jacobi clustering. We have found two basic frames associated to each spin basis: the *spin frame* and the *dynamical body frame*. Their construction is guaranteed by the fact that in the relative phase space, besides the natural existence of a Hamiltonian symmetry *left* action of $SO(3)$, it is possible to define as many Hamiltonian non-symmetry *right* actions of $SO(3)$ as the possible patterns of spin clustering.

While for $N=3$ the unique canonical spin basis coincides with a special class of global cross sections of the trivial orientation-shape $SO(3)$ principal bundle, for $N \geq 4$ the existing *spin bases* and *dynamical body frames* turn out to be unrelated to the local cross sections of the *static* non-trivial orientation-shape $SO(3)$ principal bundle, and *evolve* in a dynamical way dictated by the equations of motion. In this new formulation *both* the orientation variables and the angular velocities become, by construction, *measurable* quantities in each canonical spin basis.

For each N , every allowed spin basis provides a physically well-defined separation between *rotational* and *vibrational* degrees of freedom. The non-Abelian nature of the rotational symmetry implies that there is no unique separation of *absolute rotations* and *relative motions*. The unique *body frame* of rigid bodies is replaced here by a discrete number of *evolving dynamical body frames* and of *spin canonical bases*, both of which are grounded in patterns of spin couplings, direct analog of the coupling of quantum angular momenta.

This study of relativistic kinematics for the N -body system has been completed [69] by evaluating the rest-frame Dixon multipoles [70] and then by analyzing the role of Dixon's multipoles for open subsystems. The basic technical tool is the standard definition of the *energy momentum tensor* of the N positive-energy *free* particles on the Wigner hyperplane. On the whole, it turns out that the Wigner hyperplane is the natural framework for reorganizing a lot of kinematics connected with multipoles. Only in this way, moreover, a concept like the *barycentric tensor of inertia* can be introduced in special relativity, specifically by means of the quadrupole moments.

Finally let us remark that, as shown in Refs.[32, 55], the rest-frame instant form of dynamics automatically gives a physical ultraviolet cutoff in the spirit of Dirac and Yukawa: it is the *Møller radius* [51] $\rho = \sqrt{-\epsilon W^2}/p^2 = |\vec{S}|/\sqrt{\epsilon p^2}$ ($W^2 = -p^2 \vec{S}^2$ is the Pauli-Lubanski Casimir when $\epsilon p^2 > 0$), namely the classical intrinsic radius of the world-tube, around the covariant non-canonical Fokker-Pryce center of inertia Y^μ , inside which the non-covariance of the canonical center of mass \tilde{x}^μ is concentrated. At the quantum level ρ becomes the Compton wavelength of the isolated system multiplied its spin eigenvalue $\sqrt{s(s+1)}$, $\rho \mapsto \hat{\rho} = \sqrt{s(s+1)}\hbar/M = \sqrt{s(s+1)}\lambda_M$ with $M = \sqrt{\epsilon p^2}$ the invariant mass and $\lambda_M = \hbar/M$ its Compton wavelength. Therefore, the criticism to classical relativistic physics, based on quantum pair production, concerns the testing of distances where, due to the Lorentz signature of space-time, one has intrinsic classical covariance problems: it is

impossible to localize the canonical center of mass \tilde{x}^μ adapted to the first class constraints of the system (also named Pryce center of mass and having the same covariance of the Newton-Wigner position operator) in a frame independent way.

Let us remember [55] that ρ is also a remnant in flat Minkowski space-time of the energy conditions of general relativity: since the Møller non-canonical, non-covariant center of energy R^μ has its non-covariance localized inside the same world-tube with radius ρ (it was discovered in this way) [51], it turns out that for an extended relativistic system with the material radius smaller of its intrinsic radius ρ one has: i) its peripheral rotation velocity can exceed the velocity of light; ii) its classical energy density cannot be positive definite everywhere in every frame.

Now, the real relevant point is that this ultraviolet cutoff determined by ρ exists also in Einstein's general relativity (which is not power counting renormalizable) in the case of asymptotically flat space-times, taking into account the Poincaré Casimirs of its asymptotic ADM Poincaré charges (when supertranslations are eliminated with suitable boundary conditions).

Moreover, the extended Heisenberg relations of string theory [71], i.e. $\Delta x = \frac{\hbar}{\Delta p} + \frac{\Delta p}{T_{cs}} = \frac{\hbar}{\Delta p} + \frac{\hbar \Delta p}{L_{cs}^2}$ implying the lower bound $\Delta x > L_{cs} = \sqrt{\hbar/T_{cs}}$ due to the $y + 1/y$ structure, have a counterpart in the quantization of the Møller radius [55]: if we ask that, also at the quantum level, one cannot test the inside of the world-tube, we must ask $\Delta x > \hat{\rho}$ which is the lower bound implied by the modified uncertainty relation $\Delta x = \frac{\hbar}{\Delta p} + \frac{\hbar \Delta p}{\hat{\rho}^2}$. This could imply that the center-of-mass canonical non-covariant 3-coordinate $\vec{z} = \sqrt{\epsilon p^2}(\vec{x} - \frac{\vec{p}}{p^0} \tilde{x}^0)$ [55] cannot become a self-adjoint operator. See Hegerfeldt's theorems (quoted in Refs.[32, 55]) and his interpretation pointing at the impossibility of a good localization of relativistic particles (experimentally one determines only a world-tube in space-time emerging from the interaction region). Since the eigenfunctions of the canonical center-of-mass operator are playing the role of the wave function of the universe, one could also say that the center-of-mass variable has not to be quantized, because it lies on the classical macroscopic side of Copenhagen's interpretation and, moreover, because, in the spirit of Mach's principle that only relative motions can be observed, no one can observe it (it is only used to define a decoupled *point particle clock*). On the other hand, if one rejects the canonical non-covariant center of mass in favor of the covariant non-canonical Fokker-Pryce center of inertia Y^μ , $\{Y^\mu, Y^\nu\} \neq 0$, one could invoke the philosophy of quantum groups to quantize Y^μ to get some kind of quantum plane for the center-of-mass description. Let us remark

that the quantization of the square root Hamiltonian done in Ref.[72] is consistent with this problematic.

In conclusion, the best set of canonical coordinates adapted to the constraints and to the geometry of Poincaré orbits in Minkowski spacetime and naturally predisposed to the coupling to canonical tetrad gravity has emerged for the electromagnetic, weak and strong interactions with matter described either by fermion fields or by relativistic particles with a definite sign of the energy.

IV. A MODEL OF ADM METRIC AND TETRAD GRAVITY.

Let us now look at general relativity taking into account what has been learned about special relativity in the previous Section. We started an attempt [54, 73, 74] to revisit classic metric gravity [54] and its ADM Hamiltonian formulation [53] to see whether it is possible to define a model of general relativity able to incorporate fields and particles and oriented to a background-independent quantization. First of all to include fermions it is natural to resolve the metric tensor in terms of cotetrad fields [73, 74] [$g_{\mu\nu}(x) = E_{\mu}^{(\alpha)}(x) \eta_{(\alpha)(\beta)} E_{\nu}^{(\beta)}(x)$; $\eta_{(\alpha)(\beta)}$ is the flat Minkowski metric in Cartesian coordinates] and to reinterpret the gravitational field as a *theory of time-like observers endowed with tetrads*, whose dynamics is controlled by the ADM action thought as a function of the cotetrad fields. The model of general relativity we are going to describe gives an idealized description of an isolated system like the solar system. It can be extended to describe astrophysical systems like our galaxy, but has no relevance for cosmology at this stage.

A. Selection of a Class of Non-Compact Space-Times where the Time Evolution is Ruled by the Weak ADM Energy.

Since the standard model of elementary particles and its extensions are a chapter of the theory of representations of the Poincaré group on the non-compact Minkowski space-time and we look for a Hamiltonian description, the mathematical pseudo-Riemannian 4-manifold M^4 introduced to describe space-time is assumed to be *non-compact* and *globally hyperbolic*. This means that it admits 3+1 splittings with foliations whose leaves are space-like Cauchy 3-surfaces assumed diffeomorphic to R^3 (so that any two points on them are joined by a unique 3-geodesic). As in special relativity these 3-surfaces are also *simultaneity surfaces*, namely a convention for the synchronization of clocks. Therefore, if τ is the mathematical time labeling these 3-surfaces, Σ_{τ} , and $\vec{\sigma}$ are 3-coordinates (with respect to an arbitrary observer, a centroid $x^{\mu}(\tau)$, chosen as origin) on them, then $\sigma^A = (\tau, \vec{\sigma})$ can be interpreted as Lorentz-scalar radar 4-coordinates and the surfaces Σ_{τ} are described by embedding functions $x^{\mu} = z^{\mu}(\tau, \vec{\sigma})$. In these coordinates the metric is $g_{AB}(\tau, \vec{\sigma}) = z_A^{\mu}(\tau, \vec{\sigma}) g_{\mu\nu}(z(\tau, \vec{\sigma})) z_B^{\nu}(\tau, \vec{\sigma})$ [$g_{AB}(\tau, \vec{\sigma}) = E_A^{(\alpha)}(\tau, \vec{\sigma}) \eta_{(\alpha)(\beta)} E_B^{(\beta)}(\tau, \vec{\sigma})$ in tetrad gravity]: differently from special relativity the $z_A^{\mu}(\tau, \vec{\sigma})$ are not tetrad fields but only transition coefficients to (radar) 4-coordinates σ^A adapted to the 3+1 splitting. While in parametrized Minkowski theories the embedding $z^{\mu}(\tau, \vec{\sigma})$ are the Lagrangian configuration variables, now the components of the 4-metric

tensor $g_{AB}(\tau, \vec{\sigma})$ [or the cotetrad field $E_A^{(\alpha)}(\tau, \vec{\sigma})$] are the configuration variables, while the allowed embeddings are determined only a posteriori after the solution of Einstein's equations. As in special relativity, the Hamiltonian description has naturally built in the tools (essentially the 3+1 splitting) to make contact with experiments in a relativistic framework, where simultaneity is a frame-dependent property. The manifestly covariant description using Einstein's equations is the natural one for the search of exact solutions, but is inadequate to describe experiments.

Other requirements [54, 74] on the Cauchy and simultaneity 3-surfaces Σ_τ induced by particle physics are:

i) Each Σ_τ must be a Lichnerowicz 3-manifold [75], namely it must admit an involution so that a generalized Fourier transform can be defined and the notion of positive and negative frequencies can be introduced (otherwise the notion of particle is missing like it happens in quantum field theory in arbitrary curved space-times [2]).

ii) Both the cotetrad fields (and the metric tensor) and the fields of the standard model of elementary particles must belong to the *same family of suitable weighted Sobolev spaces* so that simultaneously there are no Killing vector fields on the space-time (this avoids the cone-over-cone structure of singularities in the space of metrics) and no Gribov ambiguity (either gauge symmetries or gauge copies [32]) in the particle sectors; in both cases no well defined Hamiltonian description is available.

iii) The space-time must be *asymptotically flat at spatial infinity* and with boundary conditions there attained in a way independent from the direction (like it is needed to define the non-Abelian charges in Yang-Mills theory [32]). This eliminates the *supertranslations* (the obstruction to define angular momentum in general relativity) and reduces the *spi group* of asymptotic symmetries to the *ADM Poincare' group*¹². The constant ADM Poincare' generators should become the standard conserved Poincare' generators of the standard model of elementary particles when gravity is turned off and the space-time (modulo a possible renormalization of the ADM energy to subtract an infinite term coming from its dependence on both G and $1/G$) reduces to the Minkowski one. As a consequence, as shown in Ref.[54], the *admissible foliations* of the space-time must have the simultaneity surfaces Σ_τ tending in a direction-independent way to Minkowski space-like hyper-planes at spatial infinity, where

¹² This group has 10 generators (Noether constants) given in the form of either 10 *strong* Poincare' charges (defined as a flux through the surface at spatial infinity) or 10 *weak* Poincare' charges (defined as volume integrals over Σ_τ) differing from the strong ones by integrals of the secondary constraints.

they must be orthogonal to the ADM 4-momentum ¹³. But, in absence of matter, these are the conditions satisfied by the Christodoulou-Klainermann space-times [76], which are near Minkowski space-time in a norm sense and have a *rest-frame* condition of zero ADM 3-momentum. Therefore the surfaces Σ_τ define the *rest frame* of the τ -slice of the universe and in this model there are asymptotic inertial observers to be identified with astronomers' *fixed stars* (the standard origin of rotations to study the precession of gyroscopes in space).

As a consequence in this class of space-times there is an *asymptotic Minkowski metric* (asymptotic background), which allows to define weak gravitational field configurations *without splitting the metric* in a background one plus a perturbation and without being a bimetric theory of gravity.

As shown in Ref.[54] these properties are concretely enforced by using a technique introduced by Dirac [11] for the selection of space-times admitting asymptotically flat 4-coordinates at spatial infinity. As a consequence the admissible embeddings of the simultaneity leaves Σ_τ have the following direction-independent limit at spatial infinity: $z^\mu(\tau, \vec{\sigma}) = X^\mu(\tau) + F^\mu(\tau, \vec{\sigma}) \rightarrow_{|\vec{\sigma}| \rightarrow \infty} X^\mu_{(\infty)}(0) + \epsilon^\mu_A \sigma^A = X^\mu_{(\infty)}(\tau) + \epsilon^\mu_r \sigma^r$. Here $X^\mu_{(\infty)}(\tau) = X^\mu_{(\infty)}(0) + \epsilon^\mu_\tau \tau$ is just the world-line of an asymptotic inertial observer having τ as proper time and ϵ^μ_A denotes an asymptotic constant tetrad with ϵ^μ_r parallel to the ADM 4-momentum (it is orthogonal to the asymptotic space-like hyper-planes). Such inertial observers corresponding to the *fixed stars* can be endowed with a spatial triad ${}^3e^r_{(a)} = \delta^r_{(a)}$, $a = 1, 2, 3$. Then the asymptotic spatial triad ${}^3e^r_{(a)}$ can be transported in a dynamical way (on-shell) by using the Sen-Witten connection [77] (it depends on the extrinsic curvature of the Σ_τ 's) in the Frauendiener formulation [78] in *every point* of Σ_τ , where it becomes a well defined triad ${}^3e^{(WSW)r}_{(a)}(\tau, \vec{\sigma})$. This defines a *local compass of inertia*, to be compared with the local gyroscopes (whether Fermi-Walker transported or not). The Wigner-Sen-Witten (WSW) local compass of inertia consists in pointing to the fixed stars with a telescope. It is needed in a satellite like Gravity Probe B to detect the frame-dragging (or gravito-magnetic Lense-Thirring effect) of the inertial frames by means of the rotation of a Fermi-Walker transported gyroscope.

Finally from Eq.(12.8) of Ref.[54] we get the set of partial differential equations for the

¹³ Incidentally, this is the first example of consistent *deparametrization* of general relativity. In presence of matter we get the description of matter in Minkowski space-time foliated with the space-like hyper-planes orthogonal to the total matter 4-momentum (Wigner hyper-planes intrinsically defined by matter isolated system). Of course, in closed space-times, the ADM Poincare' charges do not exist and the special relativistic limit is lost.

determination of the embedding $x^\mu = z^\mu(\tau, \vec{\sigma})$ (x^μ is an arbitrary 4-coordinate system in which the asymptotic hyper-planes of the Σ_τ 's have ϵ_A^μ as asymptotic tetrad): $z^\mu(\tau, \vec{\sigma}) = X_{(\infty)}^\mu(0) + F^A(\tau, \vec{\sigma}) \frac{\partial z^\mu(\tau, \vec{\sigma})}{\partial \sigma^A}$ with $F^\tau(\tau, \vec{\sigma}) = \frac{-\epsilon \tau}{-\epsilon + n(\tau, \vec{\sigma})}$ and $F^r(\tau, \vec{\sigma}) = \sigma^r + [{}^3e_{(a)}^{(WSW)r}(\tau, \vec{\sigma}) - \delta_{(a)}^r] \delta_{(a)s} \sigma^s + \frac{\epsilon n^r(\tau, \vec{\sigma})}{-\epsilon + n(\tau, \vec{\sigma})}$.

As shown in Ref.[54], a consistent treatment of the boundary conditions at spatial infinity requires the explicit separation of the *asymptotic* part of the lapse and shift functions from their *bulk* part: $N(\tau, \vec{\sigma}) = N_{(as)}(\tau, \vec{\sigma}) + n(\tau, \vec{\sigma})$, $N_r(\tau, \vec{\sigma}) = N_{(as)r}(\tau, \vec{\sigma}) + n_r(\tau, \vec{\sigma})$, with n and n_r tending to zero at spatial infinity in a direction-independent way. On the contrary, $N_{(as)}(\tau, \vec{\sigma}) = -\lambda_\tau(\tau) - \frac{1}{2} \lambda_{\tau u}(\tau) \sigma^u$ and $N_{(as)r}(\tau, \vec{\sigma}) = -\lambda_r(\tau) - \frac{1}{2} \lambda_{ru}(\tau) \sigma^u$. In the Christodoulou-Klainermann space-times [76] we have $N_{(as)}(\tau, \vec{\sigma}) = \epsilon$, $N_{(as)r}(\tau, \vec{\sigma}) = 0$.

We start off with replacement of the ten components ${}^4g_{\mu\nu}$ of the 4-metric tensor by the configuration variables of ADM canonical gravity: the *lapse* $N(\tau, \vec{\sigma}) = \epsilon + n(\tau, \vec{\sigma})$ and *shift* $N_r(\tau, \vec{\sigma}) = n_r(\tau, \vec{\sigma})$ functions and the six components of the *3-metric tensor* on Σ_τ , ${}^3g_{rs}(\tau, \vec{\sigma})$. We have ${}^4g_{AB} = \begin{pmatrix} {}^4g_{\tau\tau} = \epsilon(N^2 - {}^3g_{rs}N^rN^s) & {}^4g_{\tau s} = -\epsilon {}^3g_{su}N^u \\ {}^4g_{\tau r} = -\epsilon {}^3g_{rv}N^v & {}^4g_{rs} = -\epsilon {}^3g_{rs} \end{pmatrix}$. Einstein's equations are then recovered as the Euler-Lagrange equations of the ADM action. Besides the ten configuration variables listed above, the ADM functional phase space is *coordinated* by ten canonical momenta $\tilde{\pi}^n(\tau, \vec{\sigma})$, $\tilde{\pi}_n^r(\tau, \vec{\sigma})$, ${}^3\tilde{\Pi}^{rs}(\tau, \vec{\sigma})$ and there are eight *first class* constraints $\tilde{\pi}^n(\tau, \vec{\sigma}) \approx 0$, $\tilde{\pi}_n^r(\tau, \vec{\sigma}) \approx 0$, $\tilde{\mathcal{H}}(\tau, \vec{\sigma}) \approx 0$, ${}^3\tilde{\mathcal{H}}^r(\tau, \vec{\sigma}) \approx 0$. While the first four are *primary* constraints, the remaining four are the super-hamiltonian and super-momentum *secondary* constraints. The behavior at spatial infinity, $r = |\vec{\sigma}| \rightarrow \infty$, of the components of the 4-metric tensor and of the cotriads is $n(\tau, \vec{\sigma}) \rightarrow O(r^{-(2+\epsilon)})$, $n_r(\tau, \vec{\sigma}) \rightarrow O(r^{-\epsilon})$, ${}^3g_{rs}(\tau, \vec{\sigma}) \rightarrow (1 + \frac{M}{r}) \delta_{rs} + O(r^{-3/2})$, ${}^3e_{(a)r}(\tau, \vec{\sigma}) \rightarrow (1 + \frac{M}{2r}) \delta_{(a)r} + O(r^{-3/2})$ ($\epsilon > 0$).

Instead in ADM tetrad gravity [73, 74] there are 16 configuration variables: the cotetrad fields can be parametrized in terms of $n(\tau, \vec{\sigma})$, $n_r(\tau, \vec{\sigma})$, 3 boost parameters $\varphi_{(a)}(\tau, \vec{\sigma})$, 3 angles $\alpha_{(a)}(\tau, \vec{\sigma})$ and cotriads $e_{(a)r}(\tau, \vec{\sigma})$ on Σ_τ [$a = 1, 2, 3$]. There are 14 first class constraints $\pi_n(\tau, \vec{\sigma}) \approx 0$, $\pi_{n_r}(\tau, \vec{\sigma}) \approx 0$, $\pi_{\varphi_{(a)}}(\tau, \vec{\sigma}) \approx 0$ (the generators of local Lorentz boosts), $M_{(a)}(\tau, \vec{\sigma}) \approx 0$ (the generators of local rotations), $\Theta^r(\tau, \vec{\sigma}) \approx 0$ (the generators of the changes of 3-coordinates on Σ_τ) and $\mathcal{H}(\tau, \vec{\sigma}) \approx 0$ (the super-hamiltonian constraint). Only the last four are secondary constraints.

It can be shown [54, 74] that the addition of the DeWitt surface term to the Dirac Hamiltonian (needed to make the Hamiltonian theory well defined in the non-compact case) implies that the Hamiltonian does not vanish on the constraint surface (*no frozen reduced*

phase space picture in this model of general relativity) but is proportional to the *weak ADM energy*, which governs the τ -evolution [79], so that *an effective evolution takes place in mathematical time* τ . Moreover the Hamiltonian gauge transformations generated by the super-hamiltonian constraint *do not have the Wheeler-DeWitt interpretation* (evolution in local time), but transform an admissible 3+1 splitting into another admissible one (so that all the admissible notions of simultaneity are gauge equivalent).

It follows, therefore, that the boundary conditions of this model of general relativity imply that the real Dirac Hamiltonian is

$$H_D = E_{ADM} + H_{(D)ADM} \approx E_{ADM},$$

$$H_{(D)ADM} = \int d^3\sigma \left[n \tilde{\mathcal{H}} + n_r {}^3\tilde{\mathcal{H}}^r + \lambda_n \tilde{\pi}^n + \lambda_r^{\tilde{n}} \tilde{\pi}_n^r \right] (\tau, \vec{\sigma}) \approx 0, \\ (\text{metric gravity}),$$

$$H_{(D)ADM} = \int d^3\sigma \left[n \tilde{\mathcal{H}} + n_r {}^3\tilde{\mathcal{H}}^r + \lambda_n \tilde{\pi}^n + \lambda_r^{\tilde{n}} \tilde{\pi}_n^r + \lambda_{(a)}^{\vec{\varphi}} \pi_{\varphi(a)} + \lambda_{(a)}^{\vec{\alpha}} M_{(a)} \right] (\tau, \vec{\sigma}) \approx 0, \\ (\text{tetrad gravity}), \tag{4.1}$$

where $\lambda_n(\tau, \vec{\sigma})$ and $\lambda_n^r(\tau, \vec{\sigma})$ [and $\lambda_{(a)}^{\vec{\varphi}}(\tau, \vec{\sigma})$, $\lambda_{(a)}^{\vec{\alpha}}(\tau, \vec{\sigma})$] are *arbitrary Dirac multipliers* in front of the primary constraints. The resulting hyperbolic system of Hamilton-Dirac equations has the same solutions of the non-hyperbolic system of (Lagrangian) Einstein's equations with the same boundary conditions.

The weak ADM energy, and also the other nine asymptotic weak Poincare' charges \vec{P}_{ADM} , J_{ADM}^{AB} , are *Noether constants of the motion whose numerical value has to be given as part of the boundary conditions*. The numerical value of E_{ADM} is the *mass* of the τ -slice of the universe, while J_{ADM}^{rs} gives the value of the *spin* of the universe. The weak ADM energy $E_{ADM} = \int d^3\sigma \mathcal{E}_{ADM}(\tau, \vec{\sigma})$ has a density $\mathcal{E}_{ADM}(\tau, \vec{\sigma})$, which, like every type of energy density in general relativity, is non-tensorial and gauge-dependent, because it contains variables whose evolution depends on the arbitrary Dirac multipliers.

Since, in our case, space-time is of the Christodoulou-Klainermann type [76], the ADM 3-momentum has to vanish. This implies three first class constraints

$$\vec{P}_{ADM} \approx 0, \tag{4.2}$$

which identify the *rest frame of the universe*. As shown in Ref.[54], the natural gauge fixing to these three constraints is the requirement the the ADM boosts vanish: $J_{ADM}^r \approx 0$. In this way we decouple from the universe its 3-center of mass by making a choice of the centroid $X_{(\infty)}^\mu(\tau)$, origin of the 3-coordinates on each Σ_τ , and only *relative motions* survive, recovering a Machian flavour.

B. Meaning of the Hamiltonian Gauge Transformations and of the Gauge Fixings: Extended Space-Time Laboratories.

The first class constraints are the generators of Hamiltonian gauge transformations, under which the ADM action is quasi-invariant (second Noether theorem).

The eight infinitesimal off-shell Hamiltonian gauge transformations generating the Hamiltonian gauge orbits, have the following interpretation [54]:

- i) those generated by the four primary constraints modify the lapse and shift functions: these in turn determine how densely the space-like hyper-surfaces Σ_τ are distributed in space-time and which points have the same 3-coordinates $\vec{\sigma}$ on each Σ_τ (this is also a convention about gravito-magnetism);
- ii) those generated by the three super-momentum constraints induce a transition on Σ_τ from a given 3-coordinate system to another one;
- iii) that generated by the super-hamiltonian constraint induces a transition from a given 3+1 splitting of M^4 to another, by operating normal deformations [80] of the space-like hyper-surfaces, and shows that, like in special relativity, all the admissible notions of simultaneity are gauge-equivalent;
- iv) those generated by the three rest-frame constraints (4.2) can be interpreted as a change of centroid to be used as origin of the 3-coordinates.
- v) in tetrad gravity those generated by $\pi_{\varphi(a)}(\tau, \vec{\sigma}) \approx 0$ and $M_{(a)}(\tau, \vec{\sigma}) \approx 0$ change the cotetrads with local Lorentz transformations.

As usual, to get a completely fixed Hamiltonian gauge we add four gauge fixings to the super-hamiltonian and super-momentum constraints, which fix the form of Σ_τ (i.e. the simultaneity) and a 3-coordinate system on it ¹⁴, respectively. Their τ -constancy generates

¹⁴ Since the diffeomorphism group has no canonical identity, this gauge fixing has to be done in the following way. We choose a 3-coordinate system by choosing a parametrization of the six components ${}^3g_{rs}(\tau, \vec{\sigma})$ of the 3-metric in terms of *only three* independent functions. This amounts to fix the three functional

the gauge fixings determining the lapse and shift functions, so that the 3+1 splitting is fixed. Further τ -constancy determines the Dirac multipliers. In tetrad gravity we have also to fix a cotetrad field. Therefore, a complete Hamiltonian gauge corresponds to a *extended non-inertial space-time laboratory*, which can be shown to correspond to a 4-coordinate system of the Einstein space-time (the σ^A adapted to the 3+1 splitting) on the solutions of Einstein's equations, with a fixed dynamical chrono-geometrical structure: i) a well defined simultaneity convention for the synchronization of distant clocks (the 3-spaces Σ_τ); ii) a unit of proper time in each point of Σ_τ (the lapse function); iii) a convention for gravito-magnetism (the shift functions); iv) a 3-metric in Σ_τ for measuring spatial distances; v) a 4-metric for determining the local light-cone in each point of Σ_τ and, then, the one-way velocity of light in the geometrical optic approximations (light rays along null geodesic).

C. Quasi-Shanmugadhasan Canonical Transformation and the Generalized Inertial and Tidal Effects.

The discussion of the previous Subsection shows that there are 8 (14) *arbitrary gauge variables* in metric (tetrad) gravity. In both cases we have to identify a canonical basis $r_{\bar{a}}(\tau, \vec{\sigma})$, $\pi_{\bar{a}}(\tau, \vec{\sigma})$, $\bar{a} = 1, 2$, of *Dirac observables* (DO) as the *physical degrees of freedom of the gravitational field*, i.e. a canonical basis of predictable gauge-invariant quantities satisfying deterministic Hamilton equations governed by the weak ADM energy. This can be achieved by means of a Shanmugadhasan canonical transformation adapted to 7 (13) of the 8 (14) first class constraints (not to the super-hamiltonian one), which turns out to be a *point* canonical transformation as a consequence of the form of the finite gauge transformations. As a consequence, the old momenta are linear functionals of the new ones, with the kernels determined by a set of elliptic partial differential equations. In the new canonical basis 7 (13) new momenta vanish due to the 7 (13) constraints and their 13 conjugate configuration variables are *Abelianized gauge variables*.

Since it can be shown [54] that Lichnerowicz's identification of the conformal factor of the 3-metric on Σ_τ ($\phi = [\det {}^3g]^{1/12}$) as the unknown in the super-hamiltonian constraint is

degrees of freedom associated with the diffeomorphism parameters $\xi^r(\tau, \vec{\sigma})$. For instance, a 3-orthogonal coordinate system is identified by ${}^3g_{rs}(\tau, \vec{\sigma}) = 0$ for $r \neq s$ and ${}^3g_{rr} = \phi^2 \exp(\sum_{\bar{a}=1}^2 \gamma_{r\bar{a}} r_{\bar{a}})$. Then, we impose the gauge fixing constraints $\xi^r(\tau, \vec{\sigma}) - \sigma^r \approx 0$ as a way of identifying this system of 3-coordinates with a conventional origin of the diffeomorphism group manifold.

the correct one, as a consequence the gauge variable describing the normal deformations [80] of the simultaneity surfaces Σ_τ is the momentum $\pi_\phi(\tau, \vec{\sigma})$ canonically conjugate to $\phi(\tau, \vec{\sigma})$ (and not the trace of the extrinsic curvature of Σ_τ , the so called intrinsic York time). In this way for the first time we can identify a canonical basis of non-local and non-tensorial DO $r_{\bar{a}}(\tau, \vec{\sigma})$, $\pi_{\bar{a}}(\tau, \vec{\sigma})$, which remains canonical in the class of gauges where $\pi_\phi(\tau, \vec{\sigma}) \approx 0$, even if no one knows how to solve the super-hamiltonian constraint, i.e. the Lichnerowicz equation for the conformal factor.

In order to visualize the meaning of the various types of degrees of freedom we need the construction of a *Shanmugadhasan canonical basis* of metric gravity having the following structure with (a similar basis exists for tetrad gravity)

$$\begin{array}{c}
 \begin{array}{|c|c|c|} \hline n & n_r & {}^3g_{rs} \\ \hline \tilde{\pi}^n \approx 0 & \tilde{\pi}_{\vec{n}}^r \approx 0 & {}^3\tilde{\Pi}^{rs} \\ \hline \end{array}
 \end{array}
 \longrightarrow
 \begin{array}{c}
 \begin{array}{|c|c|c|c|c|} \hline n & n_r & \xi^r & \phi & r_{\bar{a}} \\ \hline \tilde{\pi}^n \approx 0 & \tilde{\pi}_{\vec{n}}^r \approx 0 & \tilde{\pi}_r^{\mathcal{H}} \approx 0 & \pi_\phi & \pi_{\bar{a}} \\ \hline \end{array}
 \end{array}
 \longrightarrow
 \begin{array}{c}
 \begin{array}{|c|c|c|c|c|} \hline n & n_r & \xi^r & Q_{\mathcal{H}} \approx 0 & r'_{\bar{a}} \\ \hline \tilde{\pi}^n \approx 0 & \tilde{\pi}_{\vec{n}}^r \approx 0 & \tilde{\pi}_r^{\mathcal{H}} \approx 0 & \Pi_{\mathcal{H}} & \pi'_{\bar{a}} \\ \hline \end{array}
 \end{array}
 \quad (4.3)$$

It is seen that we need a sequence of two canonical transformations.

a) The first transformation replaces seven first-class constraints with as many Abelian momenta (ξ^r are the gauge parameters, namely coordinates on the group manifold, of the passive 3-diffeomorphisms generated by the super-momentum constraints) and introduces the conformal factor ϕ of the 3-metric as the configuration variable to be determined by the super-hamiltonian constraint. Note that the final gauge variable, namely the momentum π_ϕ conjugate to the conformal factor, is the only gauge variable of momentum type: it plays the role of a *time* variable, so that the Lorentz signature of space-time is made manifest by the Shanmugadhasan transformation in the set of gauge variables $(\pi_\phi; \xi^r)$. More precisely, the first canonical transformation should be called a *quasi-Shanmugadhasan* transformation, because nobody has succeeded so far in Abelianizing the super-hamiltonian constraint.

Since it is not known how to build a global atlas of coordinate charts for the group manifold of diffeomorphism groups, it is not known either how to express the ξ^r 's, π_ϕ and the DO $r_{\bar{a}}$, $\pi_{\bar{a}}$ in terms of the original ADM canonical variables. However, since the transformation (4.3) is a *point* canonical transformation, we know the inverse point canonical transformation from the form of finite gauge transformations (see Ref.[73, 74] for the case of tetrad gravity)

$${}^3g_{rs}(\tau, \vec{\sigma}) = \frac{\partial \xi^u(\tau, \vec{\sigma})}{\partial \sigma^r} \frac{\partial \xi^v(\tau, \vec{\sigma})}{\partial \sigma^s} \phi^4(\tau, \vec{\sigma}) {}^3\hat{g}_{uv}[r_{\vec{a}}(\tau, \vec{\xi}(\tau, \vec{\sigma}))],$$

$${}^3\Pi^{rs}(\tau, \vec{\sigma}) \approx \int d^3\sigma_1 \mathcal{K}_{(a)}^{rs}(\tau, \vec{\sigma}, \vec{\sigma}_1 | \vec{\xi}, \phi, r_{\vec{a}}) \pi_{(a)}(\tau, \vec{\sigma}_1), \quad (4.4)$$

where i) ${}^3\hat{g}_{rs}$ is a 3-metric with unit determinant depending only on the two independent functions $r_{\vec{a}}(\tau, \vec{\xi}(\tau, \vec{\sigma}))$; ii) $\mathcal{K}_{(a)}^{rs}$ is a kernel determined by the requirement of canonicity of the transformation.

In absence of explicit solutions of the Lichnerowicz equation, the best we can do is to construct the *quasi-Shanmugadhasan* transformation.

b) The second canonical transformation would be instead a *complete Shanmugadhasan* transformation, where $Q_{\mathcal{H}}(\tau, \vec{\sigma}) \approx 0$ would denote the Abelianization of the super-hamiltonian constraint¹⁵. The variables $n, n_r, \xi^r, \Pi_{\mathcal{H}}$ are the final *Abelianized Hamiltonian gauge variables*, while $r'_{\vec{a}}, \pi'_{\vec{a}}$ are the final DO.

Let us stress the important fact that the Shanmugadhasan canonical transformation is a *highly non-local* (it involves the whole 3-space) transformation: this feature has a Machian flavor, although in a non-Machian context.

D. The Hamilton-Dirac Equations, Time Evolution and the Dynamical Determination of the Allowed Notions of Simultaneity.

In a completely fixed Hamiltonian gauge *all the gauge variables* $\xi^r, \pi_{\phi}, n, n_r$ *become uniquely determined functions of the DO* $r_{\vec{a}}(\tau, \vec{\sigma}), \pi_{\vec{a}}(\tau, \vec{\sigma})$, which at this stage are four arbitrary fields. Moreover, also the (unknown) solution $\phi(\tau, \vec{\sigma})$ of the Lichnerowicz equation becomes a uniquely determined functional of the DO, and this implies that all the geometrical tensors like the 3-metric ${}^3g_{rs}(\tau, \vec{\sigma})$, the extrinsic curvature ${}^3K_{rs}(\tau, \vec{\sigma})$ of the simultaneity

¹⁵ If $\tilde{\phi}[r_{\vec{a}}, \pi_{\vec{a}}, \xi^r, \pi_{\phi}]$ is the solution of the Lichnerowicz equation, then $Q_{\mathcal{H}} = \phi - \tilde{\phi} \approx 0$. Other forms of this canonical transformation should correspond to the extension of the York map [81] to asymptotically flat space-times: in this case the momentum conjugate to the conformal factor would be just York time and one could add the maximal slicing condition as a gauge fixing. Again, however, nobody has been able so far to build a York map explicitly.

surfaces Σ_τ , and the 4-metric ${}^4g_{AB}(\tau, \vec{\sigma})$ become uniquely determined functionals of the DO only.

This is true in particular for the *weak ADM energy* $E_{ADM} = \int d^3\sigma \mathcal{E}_{ADM}(\tau, \vec{\sigma})$, since the energy density $\mathcal{E}_{ADM}(\tau, \vec{\sigma})$ depends not only on the DO but also on ϕ and on the gauge variables ξ^r and π_ϕ . In a fixed gauge we get $E_{ADM} = \int d^3\sigma \mathcal{E}_{ADM}^G(\tau, \vec{\sigma})$ and this becomes the functional that rules the Hamilton equations [79] for the DO in the completely fixed gauge

$$\frac{\partial r_{\bar{a}}(\tau, \vec{\sigma})}{\partial \tau} = \{r_{\bar{a}}(\tau, \vec{\sigma}), E_{ADM}\}^*, \quad \frac{\partial \pi_{\bar{a}}(\tau, \vec{\sigma})}{\partial \tau} = \{\pi_{\bar{a}}(\tau, \vec{\sigma}), E_{ADM}\}^*, \quad (4.5)$$

where the $\{\cdot, \cdot\}^*$ are Dirac Brackets. By using the inversion of the first set of Eqs.(4.5) to get $\pi_{\bar{a}} = \pi_{\bar{a}}[r_{\bar{b}}, \frac{\partial r_{\bar{b}}}{\partial \tau}]$, we arrive at the second order in time equations $\frac{\partial^2 r_{\bar{a}}(\tau, \vec{\sigma})}{\partial \tau^2} = F_{\bar{a}}[r_{\bar{b}}(\tau, \vec{\sigma}), \frac{\partial r_{\bar{b}}(\tau, \vec{\sigma})}{\partial \tau}, \text{spatial gradients of } r_{\bar{b}}(\tau, \vec{\sigma})]$, where the $F_{\bar{a}}$'s are *effective forces* whose functional form depends on the gauge.

These Hamilton-Dirac equations have a well posed Cauchy problem as a consequence of the use of Dirac constraint theory in the Hamiltonian framework. Instead at the configurational level the Cauchy problem for Einstein's equations and the identification of the predictable quantities (DO) are so complicated that their modern treatment [82] simulates the Hamiltonian strategy.

Thus, once we have chosen any surface of the foliation as initial Cauchy surface Σ_{τ_o} and assigned the initial data $r_{\bar{a}}(\tau_o, \vec{\sigma})$, $\pi_{\bar{a}}(\tau_o, \vec{\sigma})$ of the DO, we can calculate the solution of the Einstein-Hamilton equations corresponding to these initial data.

This identifies an Einstein space-time including its dynamical chrono-geometrical structure *including the associated admissible dynamical definitions of simultaneity, distant clocks synchronization and gravito-magnetism*.

The admissible dynamical simultaneity notions in our class of space-times are much less in number than the non-dynamical admissible simultaneity notions in special relativity: as shown in Section VIII of Ref.[74], if Minkowski space-time is thought of as a special solution (with vanishing DO) of Einstein-Hamilton equations, then its allowed 3+1 splittings must have 3-conformally flat simultaneity 3-surfaces (due to the vanishing of the DO the Cotton-York tensor of Σ_τ vanishes), a restriction absent in special relativity considered as an autonomous theory.

Let us stress that the subdivision of canonical variables in two sets (gauge variables and DO) is a peculiar outcome of the quasi-Shanmugadhasan canonical transformation which

has no simple counterpart within the *Lagrangian viewpoint* at the level of the Hilbert action and/or of Einstein's equations. This subdivision amounts to an extra piece of (non-local) information which should be added to the traditional wisdom of the *equivalence principle* asserting the local impossibility of distinguishing gravitational from inertial effects. Indeed, it allows to distinguish and visualize which aspects of the local physical effects on test matter contain a *genuine gravitational* component (think to the geodesic deviation equation) and which aspects depend solely upon the choice of the *global non-inertial space-time laboratory* with the associated atlas of 4-coordinate systems in a topologically trivial space-time: these latter effects could then be named *inertial*, in analogy with what happens in the non-relativistic Newtonian case in *global rigid* non-inertial reference frames. This interpretation is possible because the Hamiltonian point of view leads naturally to a re-reading of geometrical features in terms of the traditional concept of *force*. As a consequence, we can say that in a completely fixed Hamiltonian gauge the 8 (14) gauge variables describe *generalized inertial effects* and the DO describe *generalized tidal effects* seen by the associated extended non-inertial space-time laboratory.

Let us also remark that the *reference standards* of time and length correspond to units of *coordinate time and length* and not to proper times and proper lengths [48]: this is not in contradiction with general covariance, because an extended *laboratory*, in which one defines the reference standards, corresponds to a particular *completely fixed on-shell Hamiltonian gauge* plus a local congruence of time-like observers.

The picture we have presented is not altered by the presence of matter. The only new phenomenon besides the above purely gravitational, *inertial and tidal* effects, is that from the solution of the super-hamiltonian and super-momentum constraints emerge *action-at-a-distance Newtonian-like and gravito-magnetic* effects among matter elements.

E. The Hole Argument and the Physical Identification of Point-Events.

In Ref.[83] there is a review of the various implications of Einstein's Hole Argument, a consequence of the *general covariance* of the theory in its various forms: i) invariance of the Hilbert action under passive diffeomorphisms (general coordinate transformations); ii) quasi-invariance of the ADM action under passive Hamiltonian gauge transformations; iii) invariance of Einstein's equations under active diffeomorphisms (see Ref.[84] for their passive re-interpretation). Its consequences are: i) the absence of determinism (only two of

Einstein's equations contain dynamical information: four are restrictions on initial data and four are void due to Bianchi identities), i.e. the presence of arbitrary gauge variables; ii) absence of a physical individuation of the mathematical points of the mathematical pseudo-Riemannian 4-manifold M^4 as physical point-events of space-time. In Refs.[83, 85], final re-elaboration of Refs.[86], there is a complete study and a solution of the interpretational problems connected with the Hole Argument, which, even if obsolete in physics, is still source of an open debate on the ontology of space-time in philosophy of science (see for instance Ref.[87]).

In absence of space-time symmetries, Stachel [5] suggested to identify the point-events by means of the Bergmann-Komar *intrinsic pseudo-coordinates* [88] (used as individuating fields), i.e. as four suitable functions $\bar{\sigma}^{\bar{A}}(\sigma) = F^{\bar{A}}[\Lambda_W^{(k)}[{}^4g(\sigma), \partial^4g(\sigma)]]$, ($\bar{A} = 1, 2, \dots, 4$) of the four invariant scalar eigenvalues $\Lambda_W^{(k)}(\tau, \vec{\sigma})$, $k = 1, \dots, 4$, of the Weyl tensor.

Since the Weyl eigenvalues do not depend on the lapse and shift functions, in a completely fixed gauge G they are functions only of the DO: $\Lambda_W^{(k)}(\tau, \vec{\sigma})|_G = \tilde{\Lambda}_W^{(k)}[{}^3g(\tau, \vec{\sigma}), {}^3\Pi(\tau, \vec{\sigma})]|_G = \Lambda_G^{(k)}[r_{\bar{a}}(\tau, \vec{\sigma}), \pi_{\bar{a}}(\tau, \vec{\sigma})]$

The space-time points, *mathematically individuated* by the quadruples of real numbers σ^A , corresponding to a *completely arbitrary mathematical* radar coordinate system $\sigma^A \equiv [\tau, \sigma^a]$ adapted to the Σ_τ surfaces, become now *physically individuated point-events* through the imposition of the following gauge fixings to the four secondary constraints

$$\bar{\chi}^A(\tau, \vec{\sigma}) \stackrel{def}{=} \sigma^A - \bar{\sigma}^{\bar{A}}(\tau, \vec{\sigma}) = \sigma^A - F^{\bar{A}}\left[\tilde{\Lambda}_W^{(k)}[{}^3g(\tau, \vec{\sigma}), {}^3\Pi(\tau, \vec{\sigma})]\right] \approx 0. \quad (4.6)$$

where the four functions $F^{\bar{A}}[\Lambda_W^{(k)}(\tau, \vec{\sigma})]$ (*physical individuating fields*) are chosen so that the $\bar{\chi}^A(\tau, \vec{\sigma})$'s satisfy the *orbit conditions* $\det|\{\bar{\chi}^A(\tau, \vec{\sigma}), \tilde{\mathcal{H}}^B(\tau, \vec{\sigma}')\}| \neq 0$ with $\tilde{\mathcal{H}}^B(\tau, \vec{\sigma}) = (\tilde{\mathcal{H}}(\tau, \vec{\sigma}); {}^3\tilde{\mathcal{H}}^r(\tau, \vec{\sigma})) \approx 0$. These conditions enforce the Lorentz signature, namely the requirement that $F^{\bar{r}}$ be a *time* variable, and imply that *the $F^{\bar{A}}$'s are not DO*.

The above gauge fixings allow in turn the determination of the four Hamiltonian gauge variables $\xi^r(\tau, \vec{\sigma})$, $\pi_\phi(\tau, \vec{\sigma})$. Then, their time constancy induces the further gauge fixings $\bar{\psi}^A(\tau, \vec{\sigma}) \approx 0$ for the determination of the remaining gauge variables, i.e., the lapse and shift functions in terms of the DO and then of the Dirac multipliers.

If, after this complete breaking of general covariance, we go to Dirac brackets, we enforce the point-events individuation in the form of the *identity* $\sigma^A \equiv \bar{\sigma}^{\bar{A}} = \tilde{F}^{\bar{A}}[r_{\bar{a}}(\tau, \vec{\sigma}), \pi_{\bar{a}}(\tau, \vec{\sigma})] = F^{\bar{A}}[\Lambda_W^{(k)}(\tau, \vec{\sigma})]|_G$, and on-shell this is a coordinate chart of the atlas of M^4 .

Summarizing, the effect of the whole procedure is that *the values of the DO, whose dependence on space (and on parameter time) is indexed by the chosen radar coordinates $(\tau, \vec{\sigma})$, reproduces precisely such $(\tau, \vec{\sigma})$ as the Bergmann-Komar intrinsic coordinates in the chosen gauge G .* In this way *mathematical points have become physical individuated point-events* by means of the highly non-local structure of the DO and each coordinate system σ^A is determined on-shell by the values of the 4 canonical degrees of freedom of the gravitational field in that gauge. This is tantamount to claiming that *the physical role and content of the gravitational field in absence of matter is just the very identification of the points of Einstein space-times into physical point-events by means of its four independent phase space degrees of freedom.* The existence of physical point-events in general relativity appears here as a synonym of the existence of the DO, i.e. of the true physical degrees of freedom of the gravitational field.

The addition of matter does not change this conclusion, because we can continue to use the gauge fixing (4.6). However, matter changes the Weyl tensor through Einstein's equations and contributes to the separation of gauge variables from DO in the quasi-Shanmugadhasan canonical transformation through the presence of its own DO. In this case we have DO both for the gravitational field and for the matter fields, which satisfy coupled Hamilton equations. Therefore, since the gravitational DO will still provide the individuating fields for point-events according to our procedure, *matter will come to influence the very physical individuation of points.*

Let us conclude by noting that the above gauge fixings induce a *coordinate-dependent non-commutative Poisson bracket structure* upon the *physical point-events* of space-time by means of the associated Dirac brackets implying $\{\tilde{F}_G^{\tilde{A}}(r_{\vec{a}}(\tau, \vec{\sigma}), \pi_{\vec{a}}(\tau, \vec{\sigma})), \tilde{F}_G^{\tilde{B}}(r_{\vec{a}}(\tau, \vec{\sigma}_1), \pi_{\vec{a}}(\tau, \vec{\sigma}_1))\}^* \neq 0$. The meaning of this structure in view of quantization is worth investigating.

F. Bergmann Observables versus Dirac Observables: a Conjecture.

Let us now consider the problem of the *observables* of the gravitational field. Two fundamental definitions of *observable* have been proposed in the literature.

1) The *Hamiltonian non-local Dirac observables* (DO) which, by construction, satisfy hyperbolic Hamilton equations of motion and are, therefore, deterministically *predictable*. In general, as already said, they are neither tensorial quantities nor invariant under the passive diffeomorphisms of M^4 (PDIQ).

2) The *configurational Bergmann observables* (BO) [89]: they are quantities defined on M^4 which not only are independent of the choice of the coordinates, i.e. they are quantities invariant under passive diffeomorphisms of M^4 (PDIQ), but are also *uniquely predictable from the initial data*, namely they are also DO.

In order to give consistency to Bergmann's multiple definition of BO and, in particular, to his (strictly speaking unproven) claim [89] about the *existence* of DO that are simultaneously BO, the following *conjecture* should be true:

A Main Conjecture: "The Darboux basis in the quasi-Shanmugadhasan canonical basis (4.3) of I *can be replaced* by a Darboux basis whose 16 variables are all PDIQ (or tetradic variables), such that four of them are simultaneous DO and BO, eight vanish because of the first class constraints, and the other 8 are coordinate-independent gauge variables."

If this conjecture is sound, it would be possible to construct an *intrinsic tensorial* Darboux basis of the Shanmugadhasan type. More precisely, we would have a family of quasi-Shanmugadhasan canonical bases in which all the variables are PDIQ and include 7 PDIQ first class constraints that play the role of momenta. It would be interesting, in particular, to check the form of the extra constraint replacing the standard super-hamiltonian constraint. In this way an *intrinsic characterization of inertial and tidal effects* would emerge. The same strategy applies to tetrad gravity.

Further strong support to the conjecture comes from Newman-Penrose formalism [90] where the basic tetradic fields (evaluated by using null tetrads suggested by the Hamiltonian formalism) are the 20 Weyl and Ricci scalars which are PDIQ by construction. While the vanishing of the Ricci scalars is equivalent to Einstein's equations (and therefore to a scalar form of the super-hamiltonian and super-momentum constraints), the 10 Weyl scalars plus 10 scalars describing the ADM momenta (restricted by the four primary constraints) should lead to the construction of a Darboux basis spanned only by PDIQ restricted by eight PDIQ first class constraints. Again, a quasi-Shanmugadhasan transformation should produce the Darboux basis of the conjecture. The problem of the phase space re-formulation of Newman-Penrose formalism is now under investigation.

Such an intrinsic basis would allow to start a new program of quantization of gravity, based on the idea of quantizing only the tidal effects (the BO) and not the inertial ones (the gauge variables), since the latter describe only the *appearances* of the phenomena. A

prototype of this quantization is under study in special relativity to arrive at a formulation of atomic physics in non-inertial systems: while for relativistic particles (and their non-relativistic limit) there are already preliminary results [91], for the inclusion of the electromagnetic field we have to find a way out from the Torre-Varadarajan no-go theorem [20], the obstruction to the Tomonaga-Schwinger formalism [47].

Moreover, if the weak ADM energy in a completely fixed Hamiltonian gauge can be expressed in terms of BO, this would help to clarify the problem of the coordinate-dependence of the energy density in general relativity, which we think is a preliminary step for a correct understanding of the cosmological constant and dark energy problems.

G. An Operational Determination of Space-Time.

Lacking solutions to Einstein's equations with matter corresponding to simple systems to be used as idealizations for a measuring apparatuses described by matter DO (hopefully also BO), a generally covariant theory of measurement as yet does not exist.

In the meanwhile let us sketch here a scheme for implementing - at least in principle - the physical individuation of points as an experimental setup and protocol for positioning and orientation.

a) A *radar-gauge* system of coordinates can be defined in a finite four-dimensional volume by means of a network of artificial spacecrafts similar to the Global Position System (GPS) [92]. Let us consider a family of spacecrafts, whose navigation is controlled from the Earth by means of the standard GPS. Note that the GPS receivers are able to determine their actual position and velocity because the GPS system is based on the advanced knowledge of the gravitational field of the Earth and of the satellites' trajectories, which in turn allows the *coordinate* synchronization of the satellite clocks. During the navigation the spacecrafts are test objects. Since the geometry of space-time and the motion of the spacecrafts are not known in advance in our case, we must think of the receivers as obtaining four, so to speak, *conventional* coordinates by operating a full-ranging protocol involving bi-directional communication to four *super-GPS* that broadcast the time of their standard a-synchronized clocks. This first step parallels the axiomatic construction of Ehlers, Pirani and Schild [6] of the *conformal structure* of space-time.

Once the spacecrafts have arrived in regions with non weak fields, like near the Sun or Jupiter, they become the (non test but with world-lines assumed known from GPS space

navigation) elements of an experimental setup and protocol for the determination of a local 4-coordinate system and of the associated 4-metric. Each spacecraft, endowed with an atomic clock and a system of gyroscopes, may be thought as a time-like observer (the spacecraft world-line assumed known) with a tetrad (the time-like vector is the spacecraft 4-velocity (assumed known) and the spatial triad is built with gyroscopes) and one of them is chosen as the origin of the radar-4-coordinates we want to define. This means that the natural framework should be tetrad gravity instead of metric gravity.

b) At this point we have to synchronize the atomic clocks by means of radar signals [93]. Since the geometry and the admissible simultaneity conventions of the solar system Einstein space-time are not known in advance, we could only lay down the lines of an approximation procedure starting from an arbitrary simultaneity convention. The spacecraft A chosen as origin (and using the proper time τ along the assumed known world-line) sends radar signals to the other spacecrafts, where they are reflected back to A . For each radar signal sent to a spacecraft B , the spacecraft A records four data: the emission time τ_o , the emission angles θ_o , ϕ_o and the absorption time τ_f . Given four admissible (see Ref.[22]) functions $\mathcal{E}(\tau_o, \theta_o, \phi_o, \tau_f)$, $\vec{\mathcal{G}}(\tau_o, \theta_o, \phi_o, \tau_f)$ the point P_B of the world-line of the spacecraft B , where the signal is reflected, is given radar coordinates $\tau_{(R)}(P_B) = \tau_o + \mathcal{E}(\tau_o, \theta_o, \phi_o, \tau_f)(\tau_f - \tau_o)$, $\vec{\sigma}_{(R)}(P_B) = \vec{\mathcal{G}}(\tau_o, \theta_o, \phi_o, \tau_f)$ and will be simultaneous (according to this convention) to the point Q on the world-line of the spacecraft A identified by $\tau|_Q = \tau_{(R)}(P_B)$ ¹⁶.

This allows establishing a *radar-gauge system of 4-coordinates* (more exactly a coordinate grid) $\sigma_{(R)}^A = (\tau_{(R)}; \sigma_{(R)}^r)$ in a finite region, with $\tau_{(R)} = \text{const}$ defining the radar simultaneity surfaces of this convention. By varying the functions \mathcal{E} , $\vec{\mathcal{G}}$ we change the simultaneity convention among the admissible ones .

Then the navigation system provides determination of the 4-velocities (time-like tetrads) of the satellites, namely of the ${}^4g_{(R)\tau\tau}(\sigma_{(R)}^A)$ component of the 4-metric in these coordinates. Then, employing test gyroscopes and light signals (i.e. only the *conformal structure*), by means of exchanges (two-ways signals) of *polarized* light it should be possible to determine how the spatial triads of the satellites are rotated with respect to the triad of the satellite chosen as origin. Once we have the tetrads ${}^4E_{(r)(\alpha)}^A(\tau_{(R)}, \vec{\sigma}_{(R)})$ in radar coordinates, we can build from them the inverse 4-metric ${}^4g_{(R)}^{AB}(\tau_{(R)}, \vec{\sigma}_{(R)}) =$

¹⁶ Einstein's simultaneity convention would correspond to $\mathcal{E} = \frac{1}{2}$ and to space-like hyper-planes as simultaneity surfaces.

${}^4E_{(r)(\alpha)}^A(\tau_{(R)}, \vec{\sigma}_{(R)}) {}^4\eta^{(\alpha)(\beta)} {}^4E_{(r)(\beta)}^B(\tau_{(R)}, \vec{\sigma}_{(R)})$, and the the 4-metric, in radar coordinates.

c) By measuring the spatial and temporal variation of ${}^4g_{(R)AB}(\sigma_{(R)}^C)$, the components of the Weyl tensor and the Weyl eigenvalues can in principle be determined.

d) Points a), b) and c) furnish *operationally* a slicing of space-time into surfaces $\tau_{(R)} = \text{const}$, a system of coordinates $\sigma_{(R)}^r$ on the surfaces, as well as a determination of the components of the metric ${}^4g_{(R)AB}(\sigma_{(R)}^C)$. The components of the Weyl tensor (= Riemann in void) and the local value of the Weyl eigenvalues, with respect to the radar-gauge coordinates $(\tau_{(R)}, \sigma_{(R)}^r)$ are also thereby determined. By assuming the validity of Einstein's theory, it is then a matter of computation:

i) To check whether Einstein's equations in these radar coordinates are satisfied. If not, this means that the chosen simultaneity $\tau_{(R)} = \text{const}$. is not in the class of the allowed dynamical notions of simultaneity of the Einstein solution describing the solar system. By changing the functions $\mathcal{E}, \vec{\mathcal{G}}$, we can put up an approximation procedure converging towards an admissible dynamical notion of simultaneity.

ii) If $(\tau_R, \vec{\sigma}_R)$ are the radar coordinates corresponding to a dynamical synchronization of clocks, we can get a numerical determination of the intrinsic coordinate functions $\bar{\sigma}_R^{\bar{A}}$ defining the radar gauge by the gauge fixings $\sigma_R^A - F^{\bar{A}}[\tilde{\Lambda}_W^{(k)}[{}^3g(\tau_R, \vec{\sigma}_R), {}^3\Pi(\tau_R, \vec{\sigma}_R)]] = \sigma_R^A - \tilde{F}_R^{\bar{A}}[r_{\bar{a}}(\tau_R, \vec{\sigma}_R), \pi_{\bar{a}}(\tau_R, \vec{\sigma}_R)] \approx 0$ built as intrinsic coordinates functions of the known eigenvalues of the Weyl tensor in the radar gauge.

This procedure of principle would close the *coordinative circuit* of general relativity, linking individuation to operational procedures.

H. Hamiltonian Linearization and Background-Independent Post-Minkowskian Gravitational Waves.

As a first application of the previous Hamiltonian formalism, in Ref.[94] we made a background-independent Hamiltonian linearization of vacuum tetrad gravity in a completely fixed *3-orthogonal* gauge obtained by adding 14 suitable gauge fixings, one of which is $\pi_\phi(\tau, \vec{\sigma}) \approx 0$. This allows to express all the geometrical quantities in terms of two pairs of canonically conjugated DO. In this gauge, which turns out to be *non-harmonic* in the weak field regime, the 3-metric on Σ_τ is *diagonal* and it corresponds to a unique 3-orthogonal

4-coordinate system on space-time (with an associated admissible 3+1 splitting with well defined simultaneity leaves) on the solutions of Hamilton-Dirac equations.

In this gauge it is possible to give a *background-independent* definition of a weak gravitational field: the DO $r_{\bar{a}}(\tau, \vec{\sigma})$, $\pi_{\bar{a}}(\tau, \vec{\sigma})$ should be slowly varying on a wavelength of the resulting post-Minkowskian gravitational wave, with the configurational DO $r_{\bar{a}}$ replacing the two polarizations of the harmonic gauges. A *Hamiltonian linearization* is defined in the following way:

i) Assuming $\ln \phi(\tau, \vec{\sigma}) = O(r_{\bar{a}})$, the Lichnerowicz equation can be linearized and for the first time a non-trivial solution for ϕ can be found. Using this solution all the other constraints and the elliptic canonicity conditions can be linearized and solved. By putting these solutions in the integrand of the weak ADM energy, we get a well defined form for the energy density in this gauge in terms of the DO, i.e. the physical degrees of freedom of the gravitational field.

ii) The resulting ADM energy is approximated with the terms *quadratic* in the DO and the resulting linearized Hamilton equations are studied and solved. It is explicitly checked that the linearized Einstein's equations are satisfied by this solution. Even if the gauge is not harmonic, the wave equation $\square r_{\bar{a}}(\tau, \vec{\sigma}) = 0$ is implied by the Hamilton equations and solutions satisfying the universe rest-frame condition are found (they cannot be transverse waves in the rest frame). These are the *post-Minkowskian background-independent gravitational waves*. The deformation patterns of a sphere of test particles induced by r_1 and r_2 are determined by studying the geodesic deviation equation. An explicit solution $r_{\bar{a}}(\tau, \vec{\sigma})$, $\pi_{\bar{a}}(\tau, \vec{\sigma})$ of the Hamilton equations for the DO is obtained and this allows to get the linearized 3-metric, the linearized lapse and shift functions and the linearized cotetrads in this 3-orthogonal gauge.

Therefore the dynamical chrono-geometrical structure of this Einstein space-time is completely determined and the embedding of the associated dynamical simultaneity convention can be rebuilt from the 3+1 splitting with the 3-spaces Σ_τ and the boundary conditions:

$$\begin{aligned}
z^\mu(\tau, \vec{\sigma}) &= x^\mu(0) + \epsilon_A^\mu \sigma^A - \\
&\quad - \epsilon_r^\mu \int_{-\infty}^1 \frac{d\lambda}{\lambda^2} \left[\frac{\sqrt{3}}{2} (\lambda \sigma^r) \sum_{\bar{a}u} \gamma_{\bar{a}u} \int d^3\sigma_1 \frac{\partial_{1u}^2 r_{\bar{a}}(\lambda \tau, \vec{\sigma}_1)}{4\pi |\lambda \vec{\sigma} - \vec{\sigma}_1|} - n_r(\lambda \tau, \lambda \vec{\sigma}) \right], \\
z_A^\mu(\tau, \vec{\sigma}) &= \frac{\partial z^\mu(\tau, \vec{\sigma})}{\partial \sigma^A}.
\end{aligned} \tag{4.7}$$

We are now studying tetrad gravity coupled to a perfect fluid described by a suitable singular Lagrangian [62]. The Hamiltonian linearization in the special 3-orthogonal gauge, together with an adaptation to our formalism of the theory of Dixon's multipoles [69, 70], will allow to find the post-Minkowskian (without any post-Newtonian approximation!) generalization of the quadrupole emission formula and the explicit form in this gauge of the action-at-a-distance Newton and gravito-magnetic potentials inside the fluid together with its tidal interactions. The resulting formalism should help to find a description of binary systems in a post-Minkowskian regime, where the post-Newtonian approximations fail. Moreover, the two-body problem of general relativity in the post-Minkowskian weak field regime will be studied by using a new semi-classical regularization of the self-energies, implying the $i \neq j$ rule like it happens in the electro-magnetic case [56]. Also tetrad gravity coupled to Klein-Gordon, electro-magnetic and Dirac fields is under investigation.

It will be explored the possibility of defining a scheme of Hamiltonian numerical relativity, based on expansions in the Newton constant G (the so called post-Minkowskian approximations), to study the strong field regime of tetrad gravity.

V. CONCLUDING REMARKS AND FUTURE DEVELOPMENTS.

In conclusion a unified scenario for special and general relativity (and their non-relativistic limit) taking into account their non-dynamical and dynamical, respectively, chrono-geometrical structures has emerged. It unifies many, often unrelated, points of views and allows the incorporation of a great body of phenomenology from experimental gravitation, space physics till atom and particle physics. In particular it allows to extend the description of physics from inertial frames to global non-rigid non-inertial frames, the only ones existing in general relativity, with a new insight on relativistic inertial forces and with the hope to arrive at a better understanding of the equivalence principle, especially after quantization. The establishment of this classical scenario was possible due the strength of the Hamiltonian formalism when a systematic use is made of Dirac-Bergmann theory of constraints. In particular we have identified a class of space-times in which it is possible to arrive at a *background-independent* description of both the gravitational field and elementary particles. It is then possible to go towards Newtonian physics either by a direct post-Newtonian approximation or first to make a post-Minkowskian approximation to special relativity then followed by the non-relativistic limit. Let us note that in the rest-frame instant form of relativistic mechanics it is possible to show that there are interacting models which are inequivalent at the special relativistic level but which admit the same Newtonian level: this shows that it is impossible to re-sum the series in $1/c^n$ of the post-Newtonian expansions.

At the classical level the main unsolved problem is to find either exact or approximate solutions of the Lichnerowicz equation (the super-hamiltonian constraint) beyond the post-Newtonian approximation. Only in this way we can have an idea of the coordinate-dependent modifications of Newton law (not to speak of the action-at-a-distance gravito-magnetic potentials) between matter elements implied by Einstein general relativity, before looking for its extensions or modifications.

The real challenge now is to see whether it is possible to define a new background-independent quantization scheme allowing to extend these classical results to the quantum regime in a way consistent with relativistic causality.

Let us delineate the lines of the going on researches.

A) Special relativity.

a) Since there is no accepted formulation of quantum mechanics in non-inertial frames, we are studying [91] a new quantization scheme for relativistic scalar particles on space-like hyper-planes with the differentially rotating coordinates (3.5), which correspond to an admissible family of non-rigid non-inertial frames, in the framework of parametrized Minkowski theories. The idea is to quantize only the physical particle degrees of freedom and not the embedding, which only describes the *appearances* of the phenomena by means of the inertial forces: the degrees of freedom of the embedding are treated as *generalized times* in a multi-temporal scheme. This framework allows to make a non-relativistic limit to Newton mechanics in both non-rigid and rigid non-inertial frames. In particular we want to show that, after the separation of the center of mass, the relative motions can be described in a way allowing to show that, after quantization, the spectral lines of atoms are the same both in inertial and non-inertial frames. In other words the inertial forces should produce only a noise over-imposed to the continuum spectrum of the center-of-mass free motion of the atom.

b) The next step is to try to quantize the electro-magnetic field on the arbitrary admissible 3+1 splittings allowed by parametrized Minkowski theories, but with the simultaneity 3-spaces Σ_τ restricted to admit a Fourier transform (Lichnerowicz 3-manifolds [75]). As already said, to arrive to its Tomonaga-Schwinger description we have to overcome the Torre-Varadarajan no-go theorem. This seems to require an ultraviolet regularization already for *free* fields. We hope to be able to use the Møller radius as a ultraviolet cutoff allowing to define a Fock space on each Σ_τ .

In particular this framework should allow to arrive to a formulation of relativistic atomic physics, whose semi-relativistic limit should provide a justification for the existing formalism [95].

c) This framework should allow a relativistic extension of the foundational problems of quantum mechanics like the entanglement of macroscopic bodies [4], where till now it is impossible to take into account Maxwell equations for the electro-magnetic field, either considering relativistic quantum mechanics of isolated systems with the preferred simultaneity of the Wigner hyper-planes of the rest-frame instant form or by considering relativistic atomic physics with the hope to arrive at relativistic Bell's inequalities. Let us note that already at the classical level every admissible notion of simultaneity, namely the definition of instantaneous 3-spaces Σ_τ , introduces an unavoidable non-locality. Also an attempt to define a *measuring apparatus* as those special wave functions in the Hilbert space of a macroscopic

body which do not spread in time and which behave as macroscopic Newtonian bodies (Ehrenfest theorem) could have a relativistic extension taking into account the classical relativistic delocalization of the center of mass connected with the Møller world-tube.

B) General Relativity

a) The study of perfect fluids plus tetrad gravity will allow to find in which regime it is still possible to linearize and solve the Lichnerowicz equation and then to get a background-independent post-Minkowskian quadrupole emission formula. It will help also to understand the Cauchy problem for a ball of fluid (a star), which till now has no formulation [82] either in Einstein or Newtonian gravity, since the surface of the ball is a free boundary. It will also allow a calculation of the post-Minkowskian rotation curves of galaxies, simulated by a ball of dust, to see the modification from the Keplerian evaluation.

b) It is now possible to study the coordinate-dependence of the gravito-magnetic effects [96] and of the time-delay of radar signals from satellites to Earth stations (effects of order $1/c^3$) [97]. The main open problem is to try to understand whether the empirical 4-coordinate grid used by NASA to describe the surroundings of the Earth in the solar system is a harmonic or a 3-orthogonal 4-coordinate system.

c) The study of the Hamiltonian 2-body problem by using a semi-classical regularization of the self-energies to get a $i \neq j$ rule in analogy to the electro-magnetic case with Grasmann-valued electric charges [56]. It should open the way to define a general relativistic harmonic oscillator as a prototype of a *dynamical clock* and to start to define a theory of measurement in terms of dynamical and not of test objects.

d) Then we will study the coupling of the electro-magnetic field to tetrad gravity, we shall try to find a regime where the Lichnerowicz equation can be solved and both the propagation of light and gravitational lensing can be studied at the Hamiltonian level. After an analogous study for the Klein-Gordon and Dirac fields, we will have all the ingredients for studying the coupling of the $SU(3) \times SU(2) \times U(1)$ standard model of elementary particles to tetrad gravity. In particular the Foldy-Wouthuysen transformation for the Dirac field coupled to tetrad gravity is under investigation.

e) The search of Bergmann observables suggests, as already said, to look at the Hamiltonian reformulation of the Newman-Penrose formalism by using a set of null tetrads natural from the point of view of canonical gravity. This would lead to an intrinsic definition of inertial and tidal effects and would open the way to an attempt to make a background- and coordinate-independent multi-temporal quantization of gravity. In it only the DO=BO

would be quantized, while the coordinate independent gauge variables (the inertial effects determining the appearances of the phenomena) would be treated as c-number generalized times. If ordering problems can be overcome (they should be less troublesome than in the standard attempts the canonical quantization of gravity), this quantization would respect relativistic causality due the presence of the 3+1 splittings of space-time and, like in the approach of Ref.[91], there would be a naturally defined physical scalar product.

f) Due to the importance of black holes, a Hamiltonian formulation of space-times with symmetries would be welcome but the cone over cone structure of singularities in the space of 4-metrics is an obstruction to formulate it. We are planning to start from canonical gravity without symmetries and to approach the cone of 4-metrics with one Killing vector, by adding by hand a set of Killing equations rewritten as Hamiltonian constraints. Preliminary calculations seems to indicate that the effect of these extra constraints is to forces the DO to become functions of the gauge variables. If this is confirmed, it would mean that in space-times with symmetries there are no independent degrees of freedom of the gravitational field but only generalized inertial effects besides eventual singularities.

g) Due to the dominance of dark energy in the present picture of an accelerating universe, it is worthwhile to study better the weak ADM energy of the gravitational field with its coordinate-dependent energy density containing terms proportional to both G and $1/G$: could it contribute in a coordinate-dependent way to the dark energy?

h) Finally Hamiltonian numerical gravity has to be developed by considering an iterative post-Minkowskian scheme based on developments in powers of Newton constant G .

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